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RELIABILITY AND MAINTAINABILITY ANALYSIS:
A CONCEPTUAL DESIGN MODEL

by

Anthony F. Czajkowski
Major, USAF

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Department of Quantitative Studies
School of Systems and Logistics
Air Force Institute of Technology
Wright-Patterson AFB, Ohio 45433

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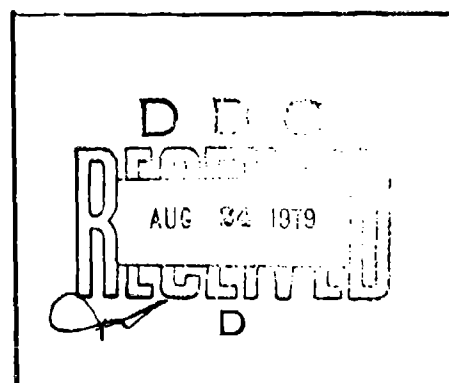
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BASIC DEFINITIONS

FAILURE: "unsatisfactory performance," usually representing a judgment of an operator or a maintenance man. This does not preclude the possibility of clear-cut failure, such as complete inoperability, in which case judgment does not enter at all. In this paper, a system/subsystem is considered to be in either a non-failed state (operating or capable of operating) or a failed state. If in a failed state, maintenance is required to return the system/subsystem to a non-failed state.

MAINTAINABILITY (M): the probability that a system/subsystem will be restored to a non-failed state within a given time when the maintenance action is performed in the prescribed manner. Maintainability is an equipment design characteristic which determines the logistics resources and the active repair time required to restore an equipment to a non-failed state. In this paper, maintainability will be expressed in terms of the system/subsystem mean-time-to-repair (MTTR).

RELIABILITY (R): the probability that a system/subsystem will perform satisfactorily for at least a given period of time when used under stated conditions. Reliability is an equipment design characteristic which determines the frequency with which a system/subsystem is down (in a failed state) for maintenance. In this paper, reliability will be expressed in terms of the system/subsystem failure rate where failure rate, λ , is the number of failures of the system/subsystem per unit time.

CHAPTER I

INTRODUCTION

The problem studied in this research is that of determining early in the development of a system, what values of reliability and maintainability should be established as system design goals and how the system goals should be apportioned to the system's subsystems so as to minimize the life cycle cost of the system. Unlike previously developed reliability and/or maintainability design decision models, the model developed in this study does explicitly allow for direct consideration of the technological and cost uncertainties which are especially prevalent during the early development phase of a system.

The method to be developed will provide a solution of the reliability/maintainability selection and apportionment problem based on the joint consideration of the following factors: (1) attainable subsystem reliability and maintainability levels are not known with certainty, (2) all life cycle cost elements (development, investment, and operating costs) are not known with certainty, (3) limited funds are available for the development of any particular system, (4) constraints on other system characteristics such as availability, weight, etc., may also exist, (5) some of the subsystem reliability and/or maintainability alternatives may be interdependent (contingent or mutually exclusive), (6) the suitability of selecting any particular combination of subsystem reliability/maintainability alternatives depends upon both life cycle cost and risk considerations. Although

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the presentation will primarily be concerned with a particular system development situation, the development of a weapon system, the model and solution technique to be developed should also have applicability to other military and non-military system developments as well.

BACKGROUND OF THE PROBLEM

A weapon system's features can be divided into performance and support characteristics, the latter into reliability and maintainability. Performance characteristics are all of the weapon system's features that contribute to its mission accomplishment, except for reliability and maintainability. Thus, aircraft performance would include measures of speed, range, payload, etc.; missile performance would include response time, payload, range accuracy, etc. Performance also includes operational reliability - the probability that an operationally ready system will react as required to accomplish its intended mission or function as planned. The support characteristics, reliability and maintainability, are all of the weapon system's features that limit the time the system is operationally ready. Unreliability results in a system being down; and maintainability determines the length of time the system is down when maintenance or servicing is required. From the logistics viewpoint, these support characteristics are the most critical features of a weapon system for together they determine both the availability of a system and the cost of the logistical resources (manpower, spares, support equipment, etc.) needed for keeping the system operationally ready throughout its useful life.

In the not too distant past, new systems were initiated, developed, produced, and introduced into the military inventory without formalized regard for their support characteristics. Our systems and equipments were designed solely to achieve operational performance - range, speed, payload, etc. - and the support characteristics - reliability and maintainability, if considered at all, were often considered after the design was so far along that significant changes could not be made. The only costs considered in making decisions were the development and initial system investment costs and the success of a weapon system development program was determined primarily on the basis of the achieved performance features. Little attention was given to the problem of supporting the system until the system was introduced into the military inventory. At that time, a determination was made of the resources that would be required to support the system and actions were then taken to provide those resources. While this system was quite inefficient, it did suffice as long as systems and equipments were relatively simple and technological change relatively slow.

In recent years, however, there has been an increasing awareness of the need to formally consider a system's support characteristics early in the design of the system; that is, during the conceptual design phase. This awareness evolved as a result of the serious difficulties that were being experienced in trying to support the increasingly complex weapon systems being introduced into the operational inventory. Increased equipment complexity, new performance requirements, and extreme environments were resulting in

higher failure rates, greater requirements for maintenance, and lower availability of current systems. Many systems were proving to be not only unreliable but also unmaintainable. The combination of these two factors was reflected in either low system availabilities or extremely high logistics support costs (estimates attribute more than half of a weapon system's life cycle cost to its logistics support costs (137)), or both.

It is important to distinguish between that case where high logistics costs for the weapon system in question result from inefficient logistics planning and management, and that case which results from ignoring reliability and maintainability during the system development process. In the first case the problem is to find more efficient ways of utilizing the resources needed to support the system given the reliability and maintainability levels designed and built or to be designed and built into the system. This problem falls within the realm of what is traditionally called logistics analyses and is primarily concerned with supporting the design given that the design decisions have already been made. In the second case the problem is to reduce the life cycle cost of a system by placing a more reliable and maintainable weapon system in the field, obtainable through better design and development of the system itself. In this case we are concerned with the technological, managerial, and financial considerations involved in designing and building reliability and maintainability into the system itself. The analysis presented here is relevant to the second case.

OBJECTIVES AND SCOPE

The objective of this research is the development of an optimizing cost model and solution technique to be used during the early planning stages of system development in the selection of quantitative system reliability and maintainability goals and the apportionment of these goals to the subsystem design levels. Development of a weapon system is the particular area of investigation.

Development of a Criterion Function

Those reliability and maintainability goals and subsystem apportionments are desired which will minimize a life cycle cost criterion function.¹ The desired performance characteristics for the system will be treated as "necessary" to the projected mission and therefore, they will be taken as given.

Since the model to be developed will explicitly treat uncertainty, an assumed certainty or simple expected value cost criterion function will not be used. Instead, the area of utility theory will be investigated in order to obtain a suitable criterion function that takes into account the military's preferences regarding costs and risk. Not only must the function reflect the military's aversion to risk and costs but it also must be operationally practicable for

¹ Solution of the model actually provides the system reliability and maintainability goals and the subsystem apportionments simultaneously. The system goals being determined by the subsystem alternatives selected.

use in the model. Hence, a mathematical representation of the criterion function must be obtained.

Development of a Selection Procedure

When numerous subsystem reliability and maintainability alternatives are being considered for selection as design goals, an unmanageable large number of possible alternative combinations may exist ((9)²⁰ possible combinations for a twenty subsystem system with nine reliability and maintainability alternatives possible for each subsystem). The selection becomes even more complex when various constraints must be observed in the selection process. The magnitude and complexity of the selection process necessitate development of a procedure which relieves the decision-maker² of the need to evaluate and establish the feasibility of numerous combinations. Such a procedure would not have to necessarily determine that a particular combination of alternatives is in any sense best but rather it would place combinations into two categories: those that the decision-maker should consider and those that do not warrant further consideration because at least one other combination of alternatives is clearly superior. The set of combinations falling into the first category constitutes a schedule of efficient combinations.

²In this paper two individuals are referred to - the analyst and the decision-maker. This clear-cut division does not exist in practice; the roles of the analyst and the decision-maker invariably overlap at least partly. However, this distinction is useful for purposes of exposition.

Use of the procedure developed for finding schedules of efficient combinations will provide for the reduction of the excessively large number of possible subsystem reliability and maintainability combinations to a small tractable set of suitable alternative combinations, each of which would be preferred for certain cost aspirations and risk aversions (as reflected in the criterion function of the model). A computerized procedure will be developed to make this reduction by finding the schedule of efficient combinations. When applied to the twenty subsystems, example presented in Chapter VI, the procedure eliminated all but 31 combinations out of a possible $(9)^{20}$ combinations for the least constrained case studied. Other, more constrained cases, had smaller numbers of combinations in the schedule of efficient combinations

Once the reduction is made and the small set of efficient combinations is obtained, the final selection is governed by the decision-maker's preferences regarding costs and risk. Recommendations to facilitate this final selection will be made.

Properties to be Included

Inputs. The unrealistic assumption will not be made that attainable subsystem reliability and maintainability alternatives and the associated future cost flows (development, investment, and operating costs) are known with certainty. Nor will the analysis be based on assumed hypothetical functional relationships between reliability and maintainability and these future cost flows. The

risk associated with a weapons development program is a result of the non-deterministic nature of these technological and cost factors, thus, these will be treated as random variables. Contractor development personnel should be able to provide useful subjective probabilistic engineering estimates for the attainable subsystem reliability and maintainability alternatives and the development and investment costs associated with them. The required operating cost estimates could be obtained from presently used contractor or military logistics planning or life cycle cost models. Recommendations to facilitate obtaining the required inputs will be discussed.

System constraints. Any practical design planning model should be able to accommodate physical limitations, e.g., weight; desired system specifications, e.g., system availability; and any budget limitations that may exist. Furthermore, the constraints must be probabilistic because of the random variable treatment of the technological and cost estimates. Therefore, the model will provide for a chance-constrained³ formulation of the constraints considered in the study.

Dependence. The model will accommodate contingent and mutually-exclusive dependence between reliability and maintainability alternatives in various subsystems. Dependence between reliability and maintainability alternatives within a subsystem will be accounted

³Chance-constrained programming has been pioneered by Charnes and Cooper (65) and developed by them and others to deal with linear programming under uncertainty.

for in the formulation of the input data used in the model.

Problem formulation. Determination of the system reliability and maintainability goals and the subsystem apportionments of these goals requires that one and only one reliability and maintainability alternative be selected for each subsystem. Fractional alternatives are disallowed and a zero or one solution, denoting rejection of selection of an alternative, is sought in which each subsystem has one alternative selected and all others rejected.

Probabilistic considerations in the constraints result in nonlinear constraints. The problem is insolvable when formulated as an integer programming problem with these nonlinear constraints. By using certain linear approximations, the constraints are linearized permitting solution of the problem as a zero-one programming problem.

Solution Technique

An algorithm by Geoffrion (128) was modified to solve the zero-one linear programming problems encountered in the selection process. The original Geoffrion algorithm was suitable for solving problems involving systems with a small number of subsystems. However, the algorithm was too slow when a large twenty subsystem problem was studied. Therefore, modification to improve the efficiency of the algorithm had to be developed. The Geoffrion algorithm and the modifications that were made to it will be discussed.

Computer Program

A computer program is written in FORTRAN IV for the

Honeywell (G.E.) 600 computer which generates schedules of efficient combinations in accordance with the prescriptions of the model. A description of the program will be presented, including input requirements and output interpretations.

Examples of Use of the Model

The utility of the model will be demonstrated by using it in the analysis of a problem involving a "large" system with twenty subsystems, each of which has nine possible reliability and maintainability alternatives. Schedules will be generated under conditions of (1) risk indifference and deterministic constraints; (2) consideration of risk and weight and availability chance-constraints; (3) consideration of risk and weight, availability, and budget chance-constraints; and (4) consideration of risk and weight, reliability, and maintainability chance-constraints. The sensitivity of schedules to various changes in the constraints will also be investigated.

ORGANIZATION OF THIS REPORT

This paper is organized into seven chapters and three appendices. Two concepts, system operational capability and system life cycle cost, which provide the conceptual basis for the model developed in this study, are discussed in Chapter II. Reliability/maintainability design decision models and logistics planning models are also discussed in Chapter II. In Chapter III the basic structure of the reliability/maintainability model is presented without

consideration of uncertainty. The adjustments required to accommodate uncertainty are introduced into the model in Chapter IV. The solution technique developed for solving the model is described in Chapter V. Chapter VI contains examples of use of the model and solution technique. The summary and recommendations are presented in Chapter VII.

The appendices are used for a listing of the solution technique computer program, a listing of the complete output from a computer run, and a listing of the input data used for the examples presented in Chapter VI.

CHAPTER II

OPERATIONAL CAPABILITY, LIFE CYCLE COST
AND RELEVANT MODELS

In the first part of this chapter, the relation of reliability and maintainability to system operational capability⁴ is discussed. Next the influence of these support characteristics on system life cycle cost is discussed. The chapter is concluded with a brief review of existing reliability and/or maintainability design decision models and logistics support planning models.

OPERATIONAL CAPABILITY

The first step in the weapons acquisition process is to define a projected mission. Once defined, the mission yields the performance characteristics desired for the system and leads to a decision either to develop a new weapon system or improve an existing one. What interests the military is acquiring an operational capability to meet the projected mission. In a perfectly general sense, the goal of the acquisition is not to develop and obtain units of hardware, but a level of operational capability. The desired level is provided by the existing technology, and any improvements accompanying the development phase of the acquisition

⁴Also referred to as "system effectiveness" in the literature.

program. Symbolically, we may define a system's operational capability as

$$OC = f(\text{Availability, Dependability, Design Adequacy})$$

where

Availability⁵ is the probability that the system, when used under stated conditions in an ideal support environment (that is available manpower, spares, equipment, etc.) will be ready to perform its assigned mission when called upon to do so.

Dependability is the conditional probability, given that a system is available, that the system will remain in operating condition for the duration of its mission. In the case of a weapon system, dependability includes not only operational reliability, but also the factors of survivability or vulnerability.

Design Adequacy is the conditional probability that a system will achieve its mission objectives, given that it is available and dependable. Design adequacy probability would have to be derived from a study of system performance characteristics, such as, speed, range, maneuverability, etc., as they relate to the particular mission under consideration.

The most reasonable form of this operational capability function appears to be a simple multiplication of the three factors, with the factors being expressed as the probabilities previously defined.

⁵ Sometimes referred to as Inherent Availability in the literature. This is the availability level designed into the system/subsystem. It excludes logistics delay time. In an actual operational environment the level of availability achieved would be less than that designed into the system/subsystem because of logistics delay time encountered in an operational support environment. This latter level of availability is termed Operational Availability. Subsequent use of the term Availability in this paper will refer to the design level or Inherent Availability of a system/subsystem.

Relation of Reliability and Maintainability
to System Operational Capability

Breaking the weapon system characteristics down into performance and support characteristics, dependability and design adequacy would be functions of the performance characteristics, while availability, would be a function of the support characteristics, reliability and maintainability.⁶ These relationships are illustrated in Figure 2.1.

The general functional relation between operational capability and a weapon's characteristics; i.e., performance, reliability, and maintainability, can be depicted as surfaces of a three-dimensional vector space (see Figure 2.2). Each surface depicts those combinations of performance P, reliability R, and maintainability M that will provide a particular level of operational capability. The surfaces slope toward the P axis for higher values of P, because the partial derivatives of the operational capability function with respect to P, M, and R, are all positive. Thus, higher levels of P are associated with lower values of R and/or M for constant operational capability. This is illustrated in Figure 2.2 by the points (P_1, R_1, M_1) and (P_0, R_0, M_0) on OC_1 .

⁶Not discussed here are other elements; such as mode of employment and the system user's operational, organizational and logistics support environment, which also affect a system's dependability, design adequacy, and availability. Since the present study is concerned primarily with system design, these other elements will not be specifically addressed in this paper.

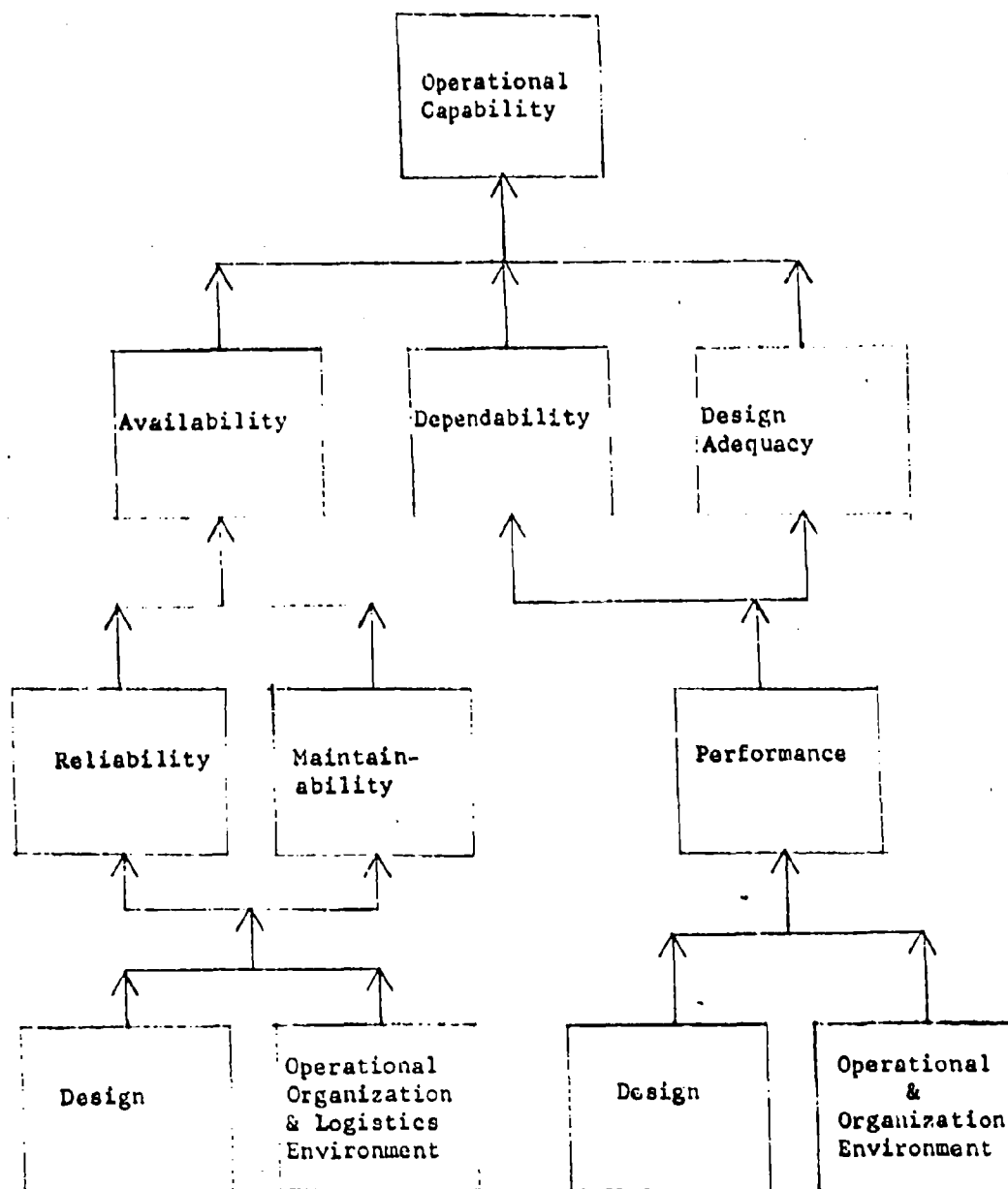


Figure 2.1

System Operational Capability Concept

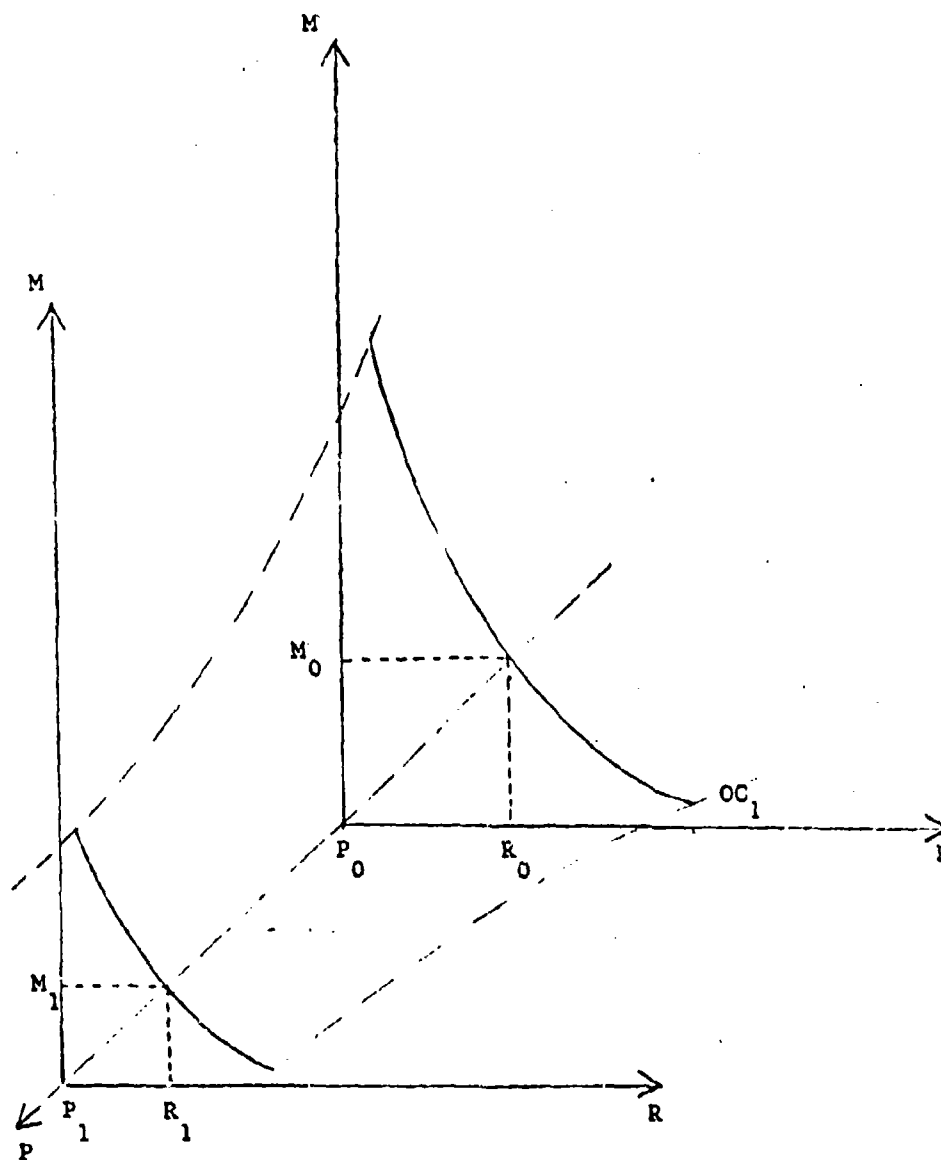


Figure 2.2

Performance-Reliability-Maintainability
Tradeoffs for $OC = OC_1$

Symbolically then, the acquisition problem is to

$$\min [C_P + C_R + C_M]$$

subject to the constraint that

$$f(P,R,M) \geq OC$$

where C_P , C_R , and C_M are costs of performance, reliability and maintainability, respectively, and OC is the desired level of operational capability being sought by the military. If the performance level is taken as given, then the acquisition problem is the choice of the proper combination of reliability and maintainability.⁷

For certain problems it is useful to discuss reliability and maintainability separately, but in achieving operational capability there is a tradeoff relationship between them. Unreliability results in a weapon system being down; and maintainability determines the length of time the system is down because of a failure and the amount of logistics resources expended to repair the failure. A system with high reliability generates relatively few down weapons per unit of operation, therefore maintainability is of lesser importance. A system that is easily maintained has

⁷performance characteristics are typically treated as "necessary" to the projected mission in a weapon's acquisition program. These characteristics are usually stated as requirements that the system to be developed must satisfy. The rationale being that a weapon system exists to accomplish some military mission. If it cannot do this it has no good reason for existence. It does not matter how supportable a system is, how reliable, how maintainable. If it cannot fulfill its mission performance requirements, it has no reason for being in the operational inventory. Therefore, the performance characteristics will be taken as being given in this study.

a relatively short average downtime and small amount of logistics resources required as a result of a failure and therefore reliability is of lesser concern. This discussion suggests that it is appropriate to view reliability and maintainability combined as a measure of system availability. Such a measure has direct meaning for a weapon system's operational capability that neither reliability nor maintainability has alone. In fact, if the performance level is given, then operational capability and availability are synonymous. Symbolically then, the acquisition problem is to

$$\min [C_P + C_R + C_M]$$

subject to

$$f_1(R, M) \geq \text{Availability}$$

$$f_2(R, M) \leq \text{Performance Imposed Physical Limitations}$$

where C_P denotes that the performance level is fixed (performance characteristics are given) and the second constraint, represents the set of constraints imposed by the required performance characteristics; e.g., the speed requirements for an aircraft would impose a weight constraint on the attainable reliability and maintainability levels.

SYSTEM LIFE CYCLE COST

A system's life cycle cost embraces all costs incurred from the initial conception of the system until the last operational unit of the system is retired from service. The costs it embraces are usually segregated into three cost categories:

1. Research and Development cost (R&D) - that is, the costs of the resources required to develop a new capability to the point where it is ready for introduction into operational use.
2. Investment costs (I) - that is, the one-time outlays required to introduce the new capability into the operational inventory. Included in this category are the costs of procuring the prime and support equipment, the initial spares, new facilities, and initial training.
3. Operating costs (O) - that is, the recurring outlays required year by year to operate and maintain the capability in service over a period of years.

An illustration of the relationship of these costs in the life of a system is depicted in Figure 2.3. Symbolically, life cycle cost is defined as

$$LCC = R\&D + I + O$$

In a system acquisition program the Life Cycle Cost concept is not an end in itself. It is not to be used solely by budgeteers and contract negotiators. Rather, it is to be used by decision-makers as the basis for making meaningful tradeoffs between development, investment, and operating costs when they consider the alternatives that must be evaluated at any decision point in an acquisition program. In essence, then, many decisions in a system development program must depend on the ability of the decision-maker and/or analyst to formulate and apply useful and realistic life cycle cost models.

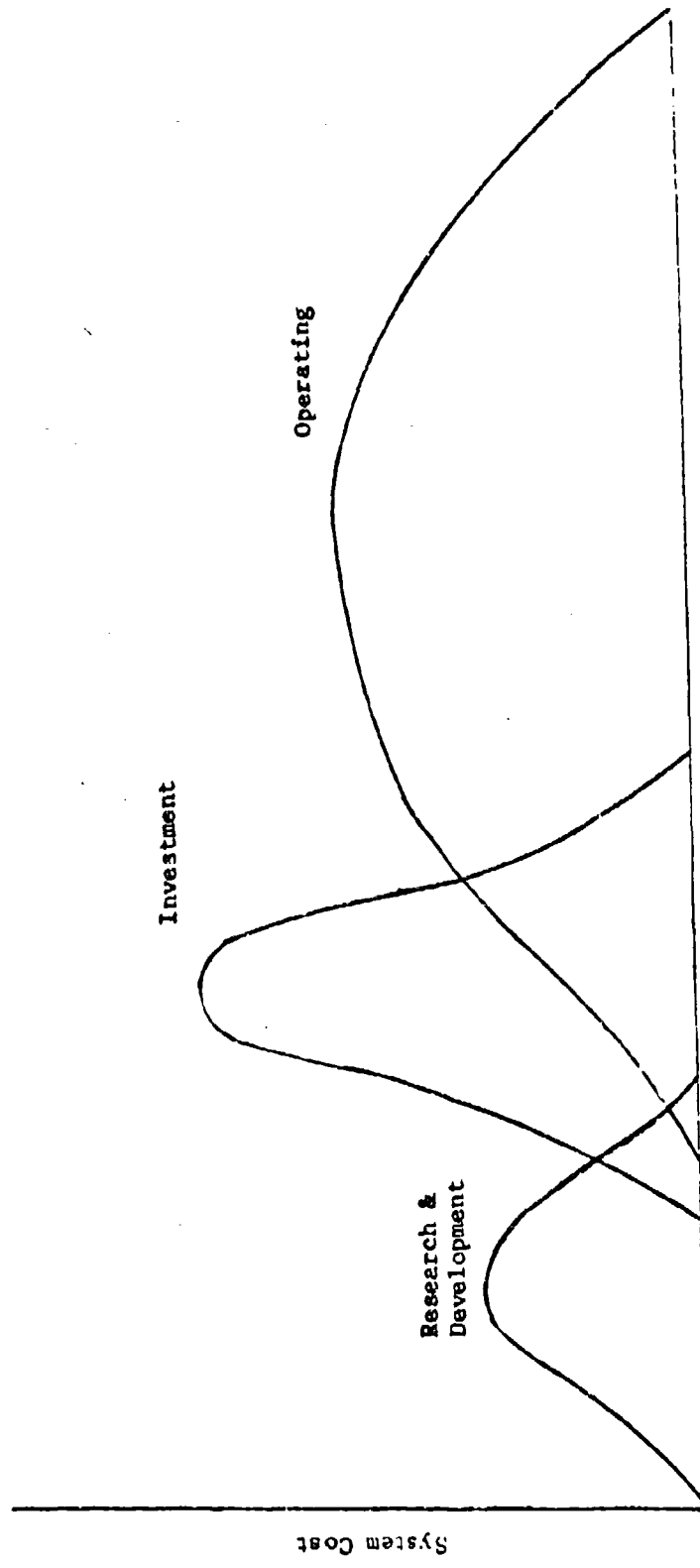


Figure 2.3
Life Cycle Cost Categories

Relation of Reliability and Maintainability
to a System's Life Cycle Cost

In the previous section of this chapter we showed that the acquisition problem, with the system performance level given, could be stated as

$$\min [C_P + C_R + C_M]$$

subject to

$$f_1(R, M) \geq \text{Availability}$$

$$f_2(R, M) \leq \text{Performance Imposed Physical Limitations}$$

where C_P , C_R , and C_M were generically defined as costs of performance, reliability and maintainability, respectively. For a fixed performance level, performance costs, C_P , in each of the three total system cost categories (Research and Development, Investment, and Operating) are also fixed.⁸ Therefore, only the relationships C_R and C_M to system life cycle cost will be examined.

Three general observations concerning the relationships between system reliability and/or maintainability and the system life cycle cost elements can be made:

1. Research and Development and Investment costs tend to increase as reliability and/or maintainability levels are improved. The rate of increase becomes pronounced for improvements substantially in excess of the levels of reliability and maintainability generally

⁸ Stating that the performance costs are fixed does not imply that they are known or can be estimated with certainty.

observed (presently being achieved).⁹

2. Within limits, Operating Costs can be reduced by improving reliability and/or maintainability. Specifically, improvements in these support characteristics reduce the maintenance portion of system operating costs.

3. The greater the desired improvement beyond presently achieved reliability and/or maintainability levels, the greater the uncertainty of achieving that improvement and the cost of doing so, hence the greater the uncertainty in all elements of system cost. The first two observations are depicted in Figure 2.4. Also illustrated in the system's life cycle cost curve. Examination of this figure reveals the fallacy of using either minimization of system development and investment costs or minimization of system operating costs as criteria for decision making in a weapons acquisition program. Minimization of total life cycle cost is the appropriate criteria and the point at which this occurs does not coincide with the minimum cost points for either development and investment costs or operating cost. What is required is the proper level of investment in each cost category so that total system life cycle cost is minimized.

The third observation indicates the importance of explicitly considering uncertainty in development programs, such as a weapon's

⁹For weapon acquisition programs concerned with system requiring large increases in performance level, significant reliability and/or maintainability improvements might be required just to achieve levels presently being achieved on systems operating at lower performance levels.

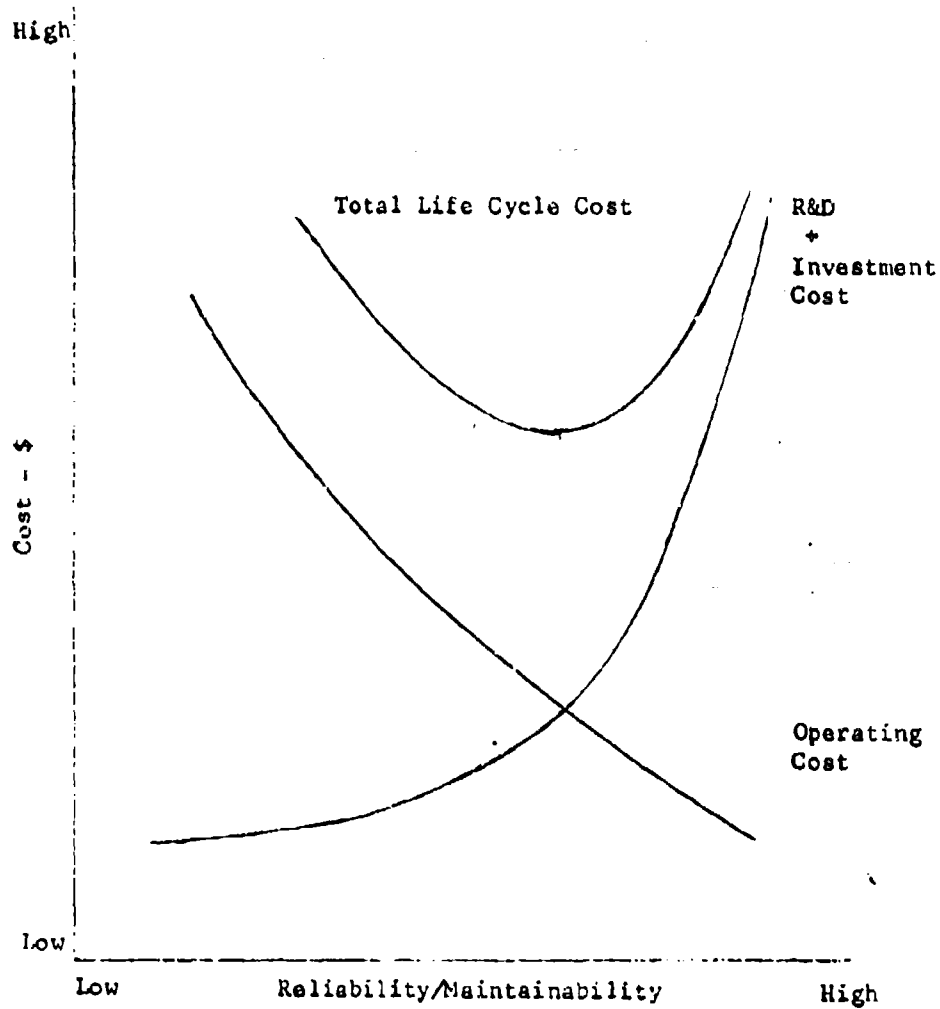


Figure 2.4

Cost Versus Reliability/Maintainability

development program, which typically require the attainment of significant reliability and maintainability improvements. Thus, decision models concerned with problems encountered in such programs must explicitly allow for uncertainty, if they are to be useful to decision-makers.

Life Cycle Costing Methods

Two basic approaches are used to derive cost estimates for life cycle costing: (1) the Cost Estimating Relationship (CER) Method and (2) the Element Estimate (EE) Method. In the CER method, costs are related to a system's performance and/or physical characteristics; for example, the development cost of an aircraft is estimated from a cost estimating relationship which relates development costs of previously developed aircraft to their weight. Or costs from one life cycle cost category are related to the costs estimated for another category; for example, a system's future support costs are estimated as being some percentage of its estimated investment cost. The CER method has two main advantages. First, it can be used early in a program because it is based on broad performance specifications and configuration concepts. A second advantage is that once developed it is not costly or difficult to use. Along with these advantages come disadvantages, the first of which is that the method cannot produce reliable results for a system which depends on new technology or substantially incorporates new design features.

The statistical relationships used are derived from experience, and that experience must be relevant to the new system. Hence the new system must fit into an existing family of systems or be similar enough to such a family to justify use of an available estimating relationship. A second disadvantage is that the models thus far established generally relate their support cost estimate to the estimated investment cost. Thus, even if an investment cost estimate increase was due to improvements in reliability and/or maintainability to be designed into the system, these models would provide an increased support cost estimate as well. This disadvantage has serious implications in that it tends to discourage tradeoffs between life cycle cost elements that would actually reduce total life cycle cost. A final disadvantage of the CER methods is that just as they are easy to use they are also easy to misuse.

In the Element Estimate (EE) Method, total cost is atomized into many elements. The elements are related in a cost structure. The structure is filled with the estimates and the life cycle cost estimate is found by summing the element estimates. The estimate for each cost element is derived from analysis of the resources (material, labor, capital) needed for the tasks which are included within the element. An example of an element structure is presented in Table 2.1. This method of life cycle costing has several advantages which make it especially suitable for reliability/maintainability tradeoff analysis. One is that it can incorporate expert input throughout. Different elements can be

Table 2.1
Life Cycle Cost Elements
For a System

I. Research and Development

- A. Preliminary design and engineering
- B. Fabrication of test equipment
- C. Test operations
- D. Miscellaneous

II. Investment

- A. Facilities
- B. Major equipment
- C. Initial inventories
- D. Initial training
- E. Miscellaneous

III. Operating Cost

- A. Equipment and facilities replacement
 - B. Maintenance
 - C. Pay and allowances
 - D. Training
 - E. Fuels, lubricants, and propellants
 - F. Miscellaneous
-

estimated by different people, and each element can be small enough to be within an individual's area of expertise. A closely related advantage is that the EE Method can be applied independently to the various subsystems of the system. A third advantage is that it allows examination of small segments of costs as well as large totals. Another advantage is that it can be in enough detail to permit study of cost differences between subsystem reliability/maintainability alternatives and reflect these differences realistically in each life cycle cost category. A final advantage of the EE Method is that it facilitates simulation because it permits individual elements to be scrutinized and it allows costs to be regrouped in numerous ways. The chief disadvantage of this costing method is that it is not as simple to use as the CER Method. Estimates must be obtained from numerous sources and more analysis is required in the EE Method.

The model developed in this study would use input estimates generated by the EE costing method. However if cost estimating relationships were derived at the subsystem level which realistically reflected the support costs versus reliability/maintainability relationships, the CER Method could also be used. Data input for the model will be discussed in Chapters III and IV.

RELEVANT MODELS

Because a model aids a decision-maker in analyzing some problems, its effectiveness is a function of a particular decision.

The reliability/maintainability area, as it interfaces with system design, concerns three broad, closely related decision situations; and a model developed to handle one phase may well also have applicability in the other phases:

1. Conceptual design - determining the system's "optimum" reliability/maintainability and apportioning the system goals to each major subsystem such that there is a "good" probability that the system goals will be met or exceeded during development.
2. Detailed system design - selecting a particular hardware design from a number of candidates.
3. Support planning - estimating the kind and quantity of resources required to support a particular design.

To illustrate these three areas, consider the reliability/maintainability of some system. In conception or early development we want to determine the system's optimum reliability/maintainability for it to achieve the mission performance requirements and to minimize life cycle costs, and the optimum reliability/maintainability of each of its subsystems. Then, given subsystem reliability/maintainability goals as well as the goals required for meeting the performance characteristic, the subsystem designer creates one or more hardware designs as candidates for the particular subsystem. He then selects that particular design which "best" meets all performance and support characteristics goals. It is possible that the same model used to establish the reliability/maintainability design goals can also be used in selecting the "best" of the subsystem design alternatives.

And finally, given that some hardware design has been selected, the logistician needs to devise his support plans for the system and its components - quantity and location of spares, how and where the item is to be repaired, support equipment needed, maintenance personnel requirements, etc.

Review of the reliability/maintainability models that have been developed over the past two decades, reveals that the major emphasis has been given to support planning model development. This is not surprising because until recent years little consideration was given to reliability/maintainability as design parameters. We only tried to optimize support of a given design and not to optimize the design for support. Of the other two model types, conceptual design and detailed design, more effort has been devoted to analysis of detailed design problems. Again this is not surprising. Initial concern for reliability in system design pertained only to operational reliability. The concern being to achieve the specified level of operational reliability in much the same manner as other performance parameters were achieved. Cost was not a constraining factor. The goal of the development program was to attain the specified performance parameters and cost efficiency was not considered to be an important factor in system development.¹⁰

Awareness of the need to consider reliability and maintainability as design parameters early in system development has evolved

¹⁰

Evidence to support this contention may be found in Peck and Scherer (37, pp 94-99).

slowly. Only since 1964¹¹ has the Department of Defense actively promulgated that this be done in weapons acquisition programs and most of the work to date has been to improve the methods of producing conceptual phase life cycle cost predictions. The studies have been along the lines of integrating conceptual phase reliability/maintainability predictions into the descriptive cost models used during conceptual planning. It is the objective of this paper to go one step beyond the descriptive cost model and develop an optimizing cost decision model. In accomplishing this objective it has been the approach to utilize previously developed cost prediction models as much as possible. Thus, the model developed in this paper is designed to utilize existing cost prediction models for its input data elements. This model begins where the existing models end.

¹¹ Since 1964 when U.S. Department of Defense Directive 4100.3 (157) was published, the services have been required to consider, estimate and evaluate the life cycle costs implied by the design decision alternatives that are required throughout the acquisition process.

CHAPTER III

BASIC STRUCTURE OF THE RELIABILITY/
MAINTAINABILITY DECISION MODEL

This chapter will be limited to the development of the basic structure of the model. The required input estimates will be identified, the system life cycle cost function will be formulated, and system/subsystem constraint functions will be developed. In this chapter no consideration will be given to the uncertain nature of the input estimates. Nor will dependence among subsystem alternatives be considered. These elements will be introduced into the model in Chapter IV.

REQUIRED MODEL INPUT ESTIMATES

Level of Detail

The development of a complex system is accomplished by division of effort. As work progresses, this division becomes more and more detailed. In a weapon's acquisition program, early conceptual feasibility studies are concerned with the system as a whole. Later in the concept phase, when the conceptual design is being created, analysis is conducted at the subsystem level of detail. A subsystem being defined as an item in a system that performs a specific function, independent of the functions of

other items, in support of the system function. During the development phase of a program the actual design and development of a subsystem is the responsibility of a design group and usually there is a separate design group for each subsystem.¹² Table 3.1 shows an example of the subsystem breakdown of a modern fighter-type aircraft.

The purpose of the conceptual design analysis is to determine a set of characteristics (both performance and support characteristics) for the subsystems such that there is a "good" probability that the system requirements will be met or exceeded. The resulting set of characteristics is called the "design point." These subsystem "design point" characteristics are used to guide design group decision-making during the course of the system development. The model developed in this paper is concerned with the establishment of the support (reliability and maintainability) "design point" characteristics during the conceptual design analysis. Thus, it is formulated at the subsystem level and all required input estimates are at the subsystem level of detail.

Input Values Required

In Chapter II we showed that when system performance characteristics are given (which is the case in a weapon's acquisition program) the acquisition problem is to

¹²In the development of a complex weapon system, such as a fighter aircraft, several contractors normally are involved with the development at the subsystem level. Thus, the subsystem design groups would actually be from several contractors. Although this paper is written as if all design groups were from the same contractor, the model developed does not depend upon this.

$$\min LCC = C_P + C_R + C_M$$

subject to

$$f_1(R, M) \geq \text{Availability}$$

$$f_2(R, M) \leq \text{Performance Imposed Physical Limitations}$$

Now we will convert this conceptual model into an operational decision model. To accomplish this we must first identify the input data elements we will use to replace the symbolic elements used in the conceptual model.

Reliability and maintainability inputs. For each subsystem, engineering estimates¹³ of attainable reliability (expressed as a mean time to failure or failure rate) and maintainability (expressed as a mean time to repair) levels as a function of proposed reliability and maintainability programs are required. These estimates could be obtained as follows: in the conceptual phase, after the subsystem performance "design point" characteristics are established but before the reliability and maintainability "design point" characteristics are established, each subsystem design group would generate several (a maximum of three is considered reasonable)¹⁴

¹³An engineering estimate is the designer's judgment of the value some parameters will attain when the subsystem development is complete and the system is operational. Engineering estimates associated with reliability/maintainability are commonly made of parameters, such as mean-time-to-failure or failure rate, mean-time-to-repair, weight, and subsystem cost.

¹⁴The feasibility of using a discrete level estimating method for obtaining reliability estimates has been documented by Colandene (166), Hevesh (174), and Fredericksen (131). These

Table 3.1

Subsystem Breakdown of
a Fighter Aircraft

SUBSYSTEM

Airframe

Landing Gear

Flight Controls

Crew Module

Propulsion

Electrical Power Supply

Lighting System

Pneudraulic Power Supply

Fuel System

Oxygen System

Instruments

Automatic Pilot

UHF Communications

IFF Communications

Radio Navigation

Fire Control System

Weapon Delivery System

Environmental Control System

different proposals; these proposals would describe the reliability/maintainability programs needed to achieve different reliability and maintainability levels (termed normal, high and ultra high for the three proposal case) for the design group's subsystem.

The programs corresponding to normal reliability/maintainability levels would be minimum reliability/maintainability programs. A minimum program may be considered as a program which provides reliability/maintainability, to the degree that current design practice in accordance with applicable specifications achieves them, without specially directed effort. Reliability/maintainability levels attainable with minimum programs would generally correspond to levels obtained in past programs. In minimum programs existing equipment/component designs would be utilized whenever possible, off-the-shelf components/parts would be used in design whenever possible, derating and redundancy would be used to the extent necessary to achieve only the operational reliability requirements, little, if any, modularization and automatic test features would be incorporated in the subsystem design, limited reliability/maintainability testing and monitoring would be conducted during development, and little Quality Control effort, in the reliability/maintainability areas, would be planned for the production phase.

papers describe the actual use of methods similar to that outlined in this paper for obtaining reliability estimates. Extension of this method to obtaining maintainability estimates should be a straight forward procedure.

The programs corresponding to high reliability/maintainability levels would be high reliability/maintainability programs. A high program may be considered as a program which attains reliability/maintainability levels which reflect a modest improvement over levels attained in the past. In high reliability/maintainability programs special designs to accommodate reliability/maintainability improvements would be created whenever possible, highly reliable components/parts would be used in design whenever possible, derating and redundancy would be used to the degree required to achieve modest improvements in subsystem reliability, limited modularization and automatic test features would be incorporated in the subsystem design, reliability/maintainability testing and monitoring programs would be initiated early in the development phase, and a Quality Control effort commensurate with the expected reliability/maintainability levels would be planned for the production phase.

The ultra high programs, corresponding to ultra high reliability/maintainability levels, may be considered as programs which would be required to achieve reliability/maintainability levels substantially higher than the levels attained in past programs. The ultra high programs would require increased activities in all of the areas proposed for the high reliability/maintainability programs.

It should be noted that the functional relations between reliability/maintainability levels and programs are not continuous relations but only discrete points representing the programs proposed

to achieve the normal, high, and ultra high reliability/maintainability levels. Between these points there are no available solutions. The only reliability/maintainability alternatives available for a subsystem are those that correspond to the possible combinations among the proposed programs. Thus, a subsystem for which there are three proposed reliability programs and three proposed maintainability would have nine reliability/maintainability alternatives. This is illustrated in matrix form in Figure 3.1.

This method of obtaining reliability/maintainability inputs by generating proposals for programs needed to achieve different reliability and maintainability levels, not only provides a systematic method for obtaining the reliability/maintainability input estimates needed for the model but facilitates use of the model for subsystem design initiation. Solution of the model not only provides reliability/maintainability "design points" for each subsystem but by referring back to the input data you also have the proposals for the reliability/maintainability programs needed to achieve the levels selected in the solution of the model. Thus, when the design group starts detailed design, they not only have "design point" goals but also planned programs which they themselves proposed for achieving the goal levels.

Cost inputs. Cost input estimates are needed for all life cycle cost elements which are dependent upon subsystem reliability and maintainability. These would include estimates of the costs of the reliability/maintainability programs proposed for attaining

Maint. Rel.	M Normal	M High	M Ultra High
R Normal	Normal	High	Ultra High
R High	Normal	High	Ultra High
R Ultra High	Normal	High	Ultra High

Figure 3.1

Three Program Reliability/Maintainability Alternatives

the different reliability/maintainability levels, those production costs which are dependent upon reliability/maintainability, the cost of initial spares and support equipment, and the recurring support (logistics) costs that are incurred during the operational life of the system.¹⁵

Estimates for the costs of the development phase reliability/maintainability efforts would be based upon the activities included in the programs proposed for achieving different reliability/maintainability levels. In fact, such cost estimates would normally be included as part of the reliability/maintainability program proposals.

Estimates of the initial spares costs, support equipment costs and recurring support costs would have to be derived for each subsystem reliability/maintainability alternative.¹⁶ Thus, nine sets of estimates would have to be derived for a typical subsystem; that is, a subsystem with nine reliability/maintainability alternatives. At first glance, derivation of these cost estimates for

¹⁵These latter cost elements (initial spares and support equipment and recurring support costs) are also dependent upon the number of units of equipment to be procured and their level of utilization. These operational factors are usually specified during the conceptual phase. Thus, values for these required operational factors will be assumed to be available for use in estimating cost elements needed for the model.

¹⁶Only those support costs that are dependent upon reliability/maintainability need to be estimated. Support costs such as training costs, tech data costs, and costs such as supply administration are generally excluded, since they probably will not vary significantly between reliability/maintainability alternatives; when their inclusion is necessary, they can be lumped together as special costs.

each subsystem reliability/maintainability alternative might appear to be a formidable task. However, there are several computerized life cycle cost models and computerized logistics support planning models available which could be used to derive these cost estimates (133, 136, 138, 142, 143, 144, 152, 153, 159, 160, 161, 162).

All these referenced models incorporate methods for determining life cycle logistics costs which could be used as input estimates for the model presently being developed. In these models, the logistics cost elements are directly related to subsystem/equipment reliability and maintainability levels. These models develop estimates of the tasks that the reliability and maintainability levels imply, and translate these into resource requirements and ultimately into dollar cost estimates. They have been designed to handle discrete input reliability and maintainability values and to provide discrete cost estimates of the resulting logistics costs. Thus, their output could be used directly as input cost estimates for the model developed in this paper.

Another feature of these models that would make them useful for deriving the logistics cost estimates needed, is that they are all computer models. Thus, even for a large multi-subsystem system with several reliability/maintainability alternatives available for each subsystem, deriving the required subsystem sets of estimates would not be too difficult a task; for example, for a twenty subsystem system with nine reliability/maintainability alternatives for each subsystem, computer derivation of the 180 sets of logistics costs estimates (initial spares and support equipment costs

and recurring support costs) required should be a relatively simple task. Also being computer based, these models can be used for handling uncertain reliability and maintainability inputs. This capability has been demonstrated by Zacks (149). He applied Monte Carlo techniques to an existing logistics model to derive probability distributions for support cost estimates.

Physical parameter inputs. The second constraint equation in the conceptual model represents physical parameter constraints imposed on the levels of reliability and/or maintainability that can be attained. In weapon system programs, physical parameter constraints exist on such parameters as weight, volume, etc. In an aircraft program, for example, system weight is a constrained parameter. This same parameter, weight, is usually a "cost" incurred in designing reliability/maintainability into a subsystem; that is, increases in subsystem reliability/maintainability levels usually require increases in subsystem weight.

In order for the reliability/maintainability decision model to accommodate physical parameter constraints, estimates are needed for the physical parameter values required for these constraints. In the model developed in this paper, weight is the only physical parameter constraint considered. (Other constraints could be added as needed). Therefore, only weight input estimates will be discussed.

For each subsystem, engineering estimates of the weight required for attaining different reliability/maintainability levels would be needed. As in the case of the reliability/maintainability

program costs, such estimates would be included as part of the reliability/maintainability program proposals.

The inputs required for the reliability/maintainability decision model are summarized in Figure 3.2. This figure shows in matrix form all input estimates required for each subsystem. All other values needed in the model are computed within the model itself; for example, subsystem availability values are computed within the model using the input reliability and maintainability values.

SYSTEM LIFE CYCLE COST FUNCTION

The objective of the optimization is to select that set of subsystem reliability/maintainability alternatives which minimizes the system life cycle cost. With the performance characteristics taken as given, the performance dependent life cycle cost elements are fixed. Therefore, we could include only the reliability/maintainability dependent life cycle cost elements in the objective function of our model. Or we could add the fixed performance related costs to our reliability/maintainability related cost elements without changing the solution obtained from the model. In this paper we will use the latter more complete life cycle cost function. Estimates of the performance related life cycle cost elements should be available during the conceptual design phase of the program. Thus, their inclusion would require little additional effort and it would avoid any confusion that might result from using a partial life cycle cost function. All subsequent discussion will assume

Rel. \ Maint.	M	M	M
	Normal	High	Ultra High
R Normal	λ_{i1}	λ_{i2}	λ_{i3}
	MTTR _{i1}	MTTR _{i2}	MTTR _{i3}
	LCC _{i1}	LCC _{i2}	LCC _{i3}
	R&D _{i1}	R&D _{i2}	R&D _{i3}
	W _{i1}	W _{i2}	W _{i3}
R High	λ_{i4}	λ_{i5}	λ_{i6}
	MTTR _{i4}	MTTR _{i5}	MTTR _{i6}
	LCC _{i4}	LCC _{i5}	LCC _{i6}
	R&D _{i4}	R&D _{i5}	R&D _{i6}
	W _{i4}	W _{i5}	W _{i6}
R Ultra High	λ_{i7}	λ_{i8}	λ_{i9}
	MTTR _{i7}	MTTR _{i8}	MTTR _{i9}
	LCC _{i7}	LCC _{i8}	LCC _{i9}
	R&D _{i7}	R&D _{i8}	R&D _{i9}
	W _{i7}	W _{i8}	W _{i9}

Figure 3.2

Input Data Matrix for Subsystem i

the life cycle cost estimates include the costs of achieving the subsystem required performance characteristics and the costs of achieving the reliability/maintainability levels.

Mathematical Formulation of the Objective Function

Let x_{ij} be the j th reliability/maintainability alternative of the i th subsystem, and LCC_{ij} be its associated estimated life cycle cost. Then, if there are N subsystem with K reliability/maintainability alternatives for each subsystem, the system life cycle cost is

$$\sum_{i=1}^N \sum_{j=1}^K LCC_{ij} x_{ij}$$

where

$$LCC_{ij} = R\&D_{ij} + I_{ij} + O_{ij}$$

and

$$x_{ij} = \begin{cases} 0 & \text{if alternative } j \text{ of subsystem } i \text{ is not selected} \\ 1 & \text{if alternative } j \text{ of subsystem } i \text{ is selected} \end{cases}$$

$$(i = 1, 2, \dots, N; j = 1, 2, \dots, K)$$

The problem to be solved by the model and solution technique is to minimize this life cycle cost function subject to the constraints which will now be discussed.

THE CONSTRAINTS

It is the purpose of the constraints of the model to set forth the basic conditions of the problem and the limitations on

each of the variables. The conditions and limitations on the given variables are determined by two set of relations. The first set of relations, which we will call the external constraints, are those set up to express desired conditions that the solution is to satisfy. External constraints involving system availability, system weight, development cost, system reliability, and system maintainability (constraints needed to accommodate dependence are discussed in Chapter IV). The model is formulated to accommodate all or only some of these external constraints. Additional external constraint can be easily added as needed. The second set, which we will call the internal constraints, are those set up to express the intra-relations of the variables within a subsystem. Internal constraints are required in the model to insure that one and only one alternative from each subsystem is in the solution.

External Constraints

System Availability. System availability, as previously defined (Chapter II, page 12), may be expressed as the fraction of total time a system is up (that is, either operating or capable of doing so, if needed),

$$\begin{aligned}
 A &= \frac{\text{TOTAL UPTIME}}{\text{TOTAL TIME}} \\
 &= \frac{\text{TOTAL TIME} - \text{DOWNTIME}}{\text{TOTAL TIME}} \\
 &= 1 - \frac{\text{DOWNTIME}}{\text{TOTAL TIME}}
 \end{aligned}$$

where

$$\text{Downtime} = \text{Number of Failures} \cdot \text{Mean Downtime/Failure}$$

where

$$\text{Number of Failures} = \lambda t_o + \lambda' t_{no}$$

$$\text{Mean Downtime/Failure} = \text{MTTR}$$

where

$$\lambda = \text{operating failure rate}$$

$$t_o = \text{total operating time}$$

$$\lambda' = \text{nonoperating failure rate}$$

$$t_{no} = \text{total nonoperating time}$$

$$\text{MTTR} = \text{mean time to repair}$$

If we assume the nonoperating failure rate, λ' , is equal to zero, then we may express the availability as

$$A = 1 - \frac{\lambda t_o \cdot \text{MTTR}}{t_o + t_{no}}$$

Let λ_i = the operating failure rate for the i th subsystem, and MTTR_i = the mean downtime per failure for the i th subsystem.

Then, the availability of the i th subsystem, A_i , may be expressed as

$$A_i = 1 - \frac{\lambda_i t_o \cdot \text{MTTR}_i}{t_o + t_{no}}$$

Now if we assume the system is available only if all subsystems are available, then, if there are N subsystems, the system availability, A_s , is

$$A_s = \prod_{i=1}^N A_i$$

$$= \prod_{i=1}^N \left[1 - \frac{\lambda_i t_o \cdot \text{MTTR}_i}{t_o + t_{no}} \right]$$

Thus, if A is the required system availability (that is, it is desired that the system have an availability greater than or equal to A), λ_{ij} is the failure rate of the j th alternative for the i th subsystem, and $MTTR_{ij}$ is the mean downtime of the j th alternative for the i th subsystem, then the system availability constraint is

$$\prod_{i=1}^N \sum_{j=1}^K \left[1 - \frac{\lambda_{ij} t_o \cdot MTTR_{ij}}{t_o + t_{no}} \right]^{x_{ij}} \geq A$$

Or, taking the natural logarithm of both sides of the inequality, we have

$$\sum_{i=1}^N \ln \sum_{j=1}^K \left[1 - \frac{\lambda_{ij} t_o \cdot MTTR_{ij}}{t_o + t_{no}} \right]^{x_{ij}} \geq \ln A$$

System weight. Performance requirements for a weapon system, such as an aircraft, restrict the amount of weight available for designing reliability/maintainability into the subsystems of the system. In principle, the possibility exists of achieving improvements in reliability/maintainability without increasing the weight of the particular subsystem involved. In practice, however, improvements in reliability/maintainability tend to increase subsystem weight.

Let W_{ij} be the weight added to the i th subsystem by the j th reliability/maintainability alternative, and W be the total weight allowed for designing reliability/maintainability into the system.

Then, the system weight constraint can be written as

$$\sum_{i=1}^N \sum_{j=1}^K w_{ij} x_{ij} \leq W.$$

System R&D cost. Constraints for system availability and system weight were the only ones included in the acquisition problem conceptual model developed in Chapter II. In practice, however, restrictions are frequently set on system R&D cost, system reliability, and system maintainability. Therefore, constraints will now be developed for these factors. In Chapter VI, the effects of imposing such additional restrictions will be demonstrated and discussed.

Let $R\&D_{ij}$ denote the development cost of the i th subsystem for the j th reliability/maintainability alternative, and $R\&D$ denote the maximum desired system development cost. Then,

$$\sum_{i=1}^N \sum_{j=1}^K R\&D_{ij} x_{ij} \leq R\&D$$

is the system development cost constraint.

System reliability. We assume that the times to failure of the subsystems are exponentially distributed.¹⁷ Let λ_s equal the failure rate of the system. Then, if there are N subsystems, λ_s

¹⁷Justification for using the exponential as the failure law of complex equipment, such as a subsystem of a weapon system, can be found in Drenick (73, pp. 680-690), Barlow & Proschan (4, pp. 109-112), Sandler (44, pp. 65-77 and pp. 109-112), and ARINC Research Corporation (51, pp. 70-75).

is equal to the sum of the subsystem failure rates. Thus, the system reliability constraint is

$$\sum_{i=1}^N \sum_{j=1}^K \lambda_{ij} x_{ij} \leq \lambda$$

where λ is the maximum desired system failure rate and λ_{ij} is the failure rate of the j th alternative for the i th subsystem.

System maintainability: The system mean time to repair, MTTR, is not the simple average of the individual subsystem values, $MTTR_i$, for, maintenance is actually performed only if a given subsystem fails. Hence, the mean maintenance time for subsystem i should be weighted by the probability that the i th subsystem fails. These weighting factors may be expressed in terms of the exponential failure rates by λ_i/λ_s so that the system mean time to repair is

$$MTTR_s = \frac{\sum_{i=1}^N \lambda_i MTTR_i}{\lambda_s}$$

where

$$\lambda_s = \sum_{i=1}^N \lambda_i$$

Thus, if we let $MTTR_{ij}$ be the mean time to repair for the j th alternative of the i th subsystem, we have as our maintainability constraint

$$\frac{\sum_{i=1}^N \sum_{j=1}^K \lambda_{ij}^{MTTR_{ij} x_{ij}}}{\sum_{i=1}^N \sum_{j=1}^K \lambda_{ij} x_{ij}} \leq MTTR$$

where MTTR is the maximum desired system mean time to repair.

Multiplying both sides of this inequality by

$$\sum_{i=1}^N \sum_{j=1}^K \lambda_{ij} x_{ij}$$

we get

$$\sum_{i=1}^N \sum_{j=1}^K \lambda_{ij}^{MTTR_{ij} x_{ij}} \leq MTTR \sum_{i=1}^N \sum_{j=1}^K \lambda_{ij} x_{ij}$$

Subtracting the right side term from both sides, we then have

$$\sum_{i=1}^N \sum_{j=1}^K \lambda_{ij}^{MTTR_{ij} x_{ij}} - MTTR \sum_{i=1}^N \sum_{j=1}^K \lambda_{ij} x_{ij} \leq 0$$

which we can rewrite as

$$\sum_{i=1}^N \sum_{j=1}^K \left[\lambda_{ij}^{MTTR_{ij} x_{ij}} - MTTR \lambda_{ij} x_{ij} \right] \leq 0$$

Finally we can factor $\lambda_{ij} x_{ij}$ from both terms in the parenthesis, so that we have

$$\sum_{i=1}^N \sum_{j=1}^K \left[\text{MTTR}_{ij} - \text{MTTR} \right] \lambda_{ij} x_{ij} \leq 0$$

as the system maintainability constraint.

Internal Constraints

The x_{ij} variables in the model represent the selection or rejection of the various subsystem reliability/maintainability alternatives (being therefore integer values, 0 or 1). The output to the model is the list of subsystem alternatives which constitute the "optimum" system. Each subsystem must be included on the list and no subsystem may be included more than once. One and only one alternative must be selected for each subsystem and all other alternatives rejected. In terms of our zero-one variables, x_{ij} , this means that in the solution each subsystem must have one x_{ij} equal to one and its remaining x_{ij} equal to zero. To insure that this occurs, each set of subsystem zero-one variables is constrained so the variables in each set sum to unity. That is,

$$\sum_{j=1}^K x_{ij} = 1, \quad i=1, \dots, N.$$

The solution algorithm developed to solve the model requires the constraints to be in inequality form. Rather than replace the N internal equality constraints with $2N$ inequalities (two inequalities for each subsystem equality), it can easily be shown that these

equality constraints hold if, and only if, the following is the case:

$$-N + \sum_{i=1}^N \sum_{j=1}^K x_{ij} \geq 0$$

$$1 - \sum_{j=1}^K x_{ij} \geq 0, \quad i=1, \dots, N.$$

Thus, we now need only $(N + 1)$ inequalities to represent the internal constraints of the problem whereas $(2N)$ inequalities would have been needed.

SUMMARY OF BASIC MODEL STRUCTURE

Combining the results of the previous sections, the reliability/maintainability conceptual design decision model can be stated as:

$$\text{minimize} \quad \sum_{i=1}^N \sum_{j=1}^K LCC_{ij} x_{ij}$$

subject to some or all of the following constraints:

$$1) \quad \sum_{i=1}^N \ln \sum_{j=1}^K \left[1 - \frac{\lambda_{ij} t_o \cdot MTTR_{ij}}{t_o + t_{nc}} \right] x_{ij} \geq \ln A$$

$$2) \quad \sum_{i=1}^N \sum_{j=1}^K w_{ij} x_{ij} \leq W$$

$$3) \quad \sum_{i=1}^N \sum_{j=1}^K R\&D_{ij} x_{ij} \leq R\&D$$

$$4) \quad \sum_{i=1}^N \sum_{j=1}^K \lambda_{ij} x_{ij} \leq \lambda$$

$$5) \quad \sum_{i=1}^N \sum_{j=1}^K \left[MTTR_{ij} - MTTR \right] \lambda_{ij} x_{ij} \leq 0$$

$$6a) \quad -N + \sum_{i=1}^N \sum_{j=1}^K x_{ij} \geq 0$$

$$6b) \quad 1 - \sum_{j=1}^K x_{ij} \geq 0, \quad i=1, \dots, N$$

The 0-1 nature of the x_{ij} variables is taken care of by the solution procedure. Thus it is not necessary to include additional constraints to insure this.

CHAPTER IV

CONSIDERATION OF UNCERTAINTY AND

DEPENDENCE IN THE MODEL

The construction of this reliability/maintainability conceptual design model is not made with the intent of uncovering any "truths" in the decision process, nor with the intent of demonstrating whether or not the decision-maker behaves in a mathematically optimum manner. The intent is, however, to provide a consistent and systematic method for handling a complex decision situation, given an approximate description of how the decision-maker does behave in the face of uncertainty (that is, he has aversion to risk and antipathy for costs).

DECISION MAKING UNDER CERTAINTY,
RISK, OR UNCERTAINTY

A decision is said to be a decision under "certainty" when, for all available alternative courses of action, each alternative is known by the decision-maker to lead to a specific outcome. The decision-maker has perfect information regarding the occurrence of outcomes. Usually decisions encountered in the weapons acquisition environment can seldom be found which qualify as decisions under certainty, particularly when any substantial aspect of futurity is involved.

In the systems development environment, information regarding the costs and potential results of the reliability and maintainability alternatives is likely to be imperfect. When the decision situation is characterized by imperfect information regarding the occurrence of outcomes, the decision is either classified as being one under risk or one under uncertainty. A decision is said to be a decision under risk when each available alternative course of action can lead to several or many outcomes, and the probabilities or likelihoods of occurrence of these outcomes are known or can be estimated. A decision under uncertainty, on the other hand, is a decision for which the probabilities or likelihoods of occurrence of these outcomes are neither known nor can they be estimated. (A good review of decision making under certainty, risk, and uncertainty can be found in Luce (31) or in Morris (35) .)

If the reliability/maintainability conceptual design problem is to be treated as a decision under risk, the probability distributions of the attainable reliability/maintainability levels and the cost flows must be obtainable. An ideal situation would be one in which the distributions are obtained empirically, using statistical procedures to compile relative frequency information contained in historical data. Unfortunately, in weapons acquisitions such objective information is seldom available to the decision-maker.

Since empirically determined probability distributions are usually unavailable, can the reliability/maintainability decision be classified as a decision under risk? The subjectivist statisticians,

for example Ramsey (41) and Savage (45) , maintain that if the decision-maker or some other knowledgeable evaluator can provide subjective or intuitive information regarding the desired probabilities, such information can be used in the same manner as objective probabilities. Luce (31) , p. 299) indicates that the decision-maker usually has at least some partial information from which the probabilities of occurrence of the outcomes can be estimated. When the decision concerns a real life problem (as opposed to a contrived exercise or game in which the decision-maker is presented the outcomes but not the information from which they were derived) Ackoff, Gupta, and Minas (1 , pp. 53-55) make an even stronger plea for treating such a decision as one under risk instead of one under uncertainty. They maintain that in order for the decision-maker to even specify the relevant outcomes of the situation, he necessarily has more information regarding the situation than can be processed in a decision under uncertainty.

If the choice were between using objective, relative frequency probability estimates (say, obtained empirically on the basis of a large number of observations) or using less reliable subjective probability estimates (say, obtained on the basis of perceptions and judgments from one or several experienced weapon designers) even the most devout subjectivist statistician would no doubt accede to the preferability of the objective estimates. However, the choice is not one of substituting more reliable (objective) probability estimates for less reliable (subjective) estimates, but rather one of substituting less reliable (subjective) estimates for no estimates

whatsoever. It is at this juncture that the classical objectivist statistician and the subjectivist statistician differ. The classical statistician denies the legitimacy of subjective information, discards it, and would proceed to treat the decision as one under uncertainty. The subjectivist statistician on the other hand defends judicious consideration of all information, even subjective when objective is not available, and contends that subjective judgments permit the decision to be made under conditions of risk.

In the pages that follow, the subjectivist approach will be adhered to in order to treat the conceptual design decision under uncertainty as though it were a case of risk.¹⁶

CHOICE CRITERIA UNDER RISK

Deciding to cast one's uncertainty model in the form of risk may help to narrow the field of alternative decision-making criteria but it does not, by itself, provide a final solution to the problem. Even admitting that one's eventual choice is likely to be probabilistic in nature, a plenitude of possibilities - and problems - remain. Let us consider two of the more popular attempts which have been made in this direction - expected value criteria and expected utility criteria.

¹⁸ Precedence for treating decision problems encountered in system development programs as decisions under risk rather than decisions under uncertainty may be found in studies by Diemmain (119) , Tinsion (145), Zachs (149) , and Seiler (47) .

Expected Value Criteria

The most common principle of choice in practice suggests that the decision-maker choose the alternative which maximizes expected gain; that is, maximizes expected profit or minimizes expected cost. Although maximizing expected gain may be shown in innumerable situations (its application is of equal validity in virtually any environment where the decision involved is repetitive in nature and possible outcomes are not extreme), to be a highly defensible decision criteria, we must reject it as a general criteria for decision-making under risk. First of all, an attempt to maximize gain under conditions of risk has long been recognized as capable of leading to ridiculous decisions, such as paying an infinite sum to play the notorious St. Petersburg game (62). Second, it is contrary to observed behavior, in that most decision-makers seek to avoid risks and are willing to forego gain in order to avoid them. An example of this is diversification of investments. A decision-maker who sought only to maximize the expected gain would never prefer a diversified investment portfolio. If one investment had greater expected gain than any other, the investor would place all his funds in this investment. If several investments had the same (greatest) expected gain, the investor would be indifferent among portfolios, diversified or not, which contained only these investments. Thus, if we consider diversification a sound principle of investment, we must reject the objective of simply maximizing expected gain. Markowitz (32) provides further arguments along these lines in his development of a model for portfolio selection of securities.

When outcomes are risky and extreme, as they are in a weapons acquisition decision problem, a preference structure is desired which reflects more than just predilection for maximizing expected gain (minimizing expected cost in a weapons program). What is needed is a preference structure that not only reflects the decision-maker's hopes to maximize expected gain but also his hopes to avoid or minimize risk. Most decision-makers have an aversion to risk and we need a criterion function that reflects this aversion.

Expressing Risk Quantitatively

"Risk" is a generic term for which intuitive connotation exist, but which is not an operationally suitable quantity without further specification. Risk, to the decision-maker, is the possibility of deviations of future outcomes from their expected values. All risk arises from such deviations; more specialized ideas of risk are derivative from risk defined in this manner.

Can risk be measured? In principle it can; whenever we can derive a probability distribution of outcomes we can measure its dispersion which will serve as a measure of risk. In practice it may be quite difficult and many decision-makers as well as leading economists have argued against it (21, 177). The principal arguments against it seem to be that so many variable factors must be considered in attempting to evaluate risk that it is either impossible, too time consuming, or too expensive to do so with enough accuracy to be of any use. A second problem is that even if we had accurate measurements, the state of theoretical development.

has been such that we would not have been able to use them operationally. It is our view that neither of these arguments remain valid. The computational argument was valid until the advent of the electronic computer; it is no longer so. The development of theory leaves a great deal to be desired. The model developed in this paper is an operational, if imperfect, means of handling measurable differences in risk in a weapons acquisition situation.

The use of measures of dispersion to represent the degree of risk associated with undertaking a particular decision is not new. Markowitz (32), in his classic security portfolio selection model, measures risk in terms of the portfolio's variance of returns. Cord (71) in his project selection model, specified implicitly that the annual income variance of the selected projects reflects the risk associated with undertaking those projects.¹⁹ Furthermore, the use of variance in this capacity is not entirely arbitrary, but has substantive justification (as an approximation) in the important area of utility theory.

Expected Utility

In a monumental work, von Neumann and Morgenstein (52) developed the essence of modern utility theory (and game theory). The von Neumann-Morgenstern theory provides the framework for obtaining a numerical representation of a preference ordering among

¹⁹ Other examples of using variance as a surrogate for risk may be found in Canada and Wadsworth (64), Edelman and Greenberg (74), Hertz (82), Hillier (83), Linter (89), Mac and Sarndal (91), Mac and Wallingford (92), Naslund (97), Wagle (108), and Watters (176).

alternatives, this representation being the "utility function."

If alternative I is preferred to alternative II, then alternative I is said to have more "utility" for the person expressing the preference than does alternative II.

Very roughly, their assumptions essentially are that the decision-maker can (1) give a consistent preference order for all alternatives of interest, and (2) express consistent preferences for combinations of alternatives and stated probabilities. Under these assumptions, von Neumann and Morgenstern show that one can introduce utility associations to the basic alternatives in such a manner that, if the decision-maker is guided solely by maximization of expected utility, he is acting in accord with his true tastes. A detailed discussion of the von Neumann-Morgenstern utility theory is given by Savage (45); implications and interpretation of their assumptions appear in numerous sources, including Luce and Raiffa (31), Fishburn (12), and Ackoff, Gupta and Minas (1).

The von Neumann-Morgenstern procedure determines a utility function which is unique only up to a linear transformation. The utility scale has neither a true zero nor a unique unit of measure and can be transformed by multiplying or dividing every value by the same positive constant and/or adding a constant to every value without altering the desired preference relationships of the utility function. A stronger measurement of utility with a true zero, commonly called a ratio scale, has been proposed by Restle (42); however, the decision situation to be considered does not require

this type of measurement. Sufficient information is contained in "combination A of subsystem alternatives has maximum expected utility" to enable accomplishment of system selection, and information such as "the expected utility of combination A is 21.2 utilities" is beyond the requirements of the model.

Properties of a utility curve. Within a given range of outcomes, the utility function can have three general shapes as shown in Figure 4.1. We are not to think of the vertical axis as representing pleasure and pain. It simply represents the degree to which the individual is willing to take risks for outcomes presented along the horizontal axis. All three functions increase monotonically throughout. However, the marginal utility of an additional dollar of gain (that is, dollar of cost decrease in our case) varies among the three cases. Function I described the attitudes of a decision-maker who has a constant marginal utility of money, indicating that he values an additional dollar of gain just as highly regardless of whether it is the first dollar of gain or the 100,001st dollar of gain. Such a decision-maker is indifferent to risk and if he acts consistently, he will want to choose so as to minimize expected dollar cost. For him, the utility of money is proportional to the amount, and the expected utility is proportional to the expected gain.

Function II describes the attitudes of a decision-maker who has a decreasing marginal utility of money, indicating that as dollar gains increase (cost decrease), additional gains become

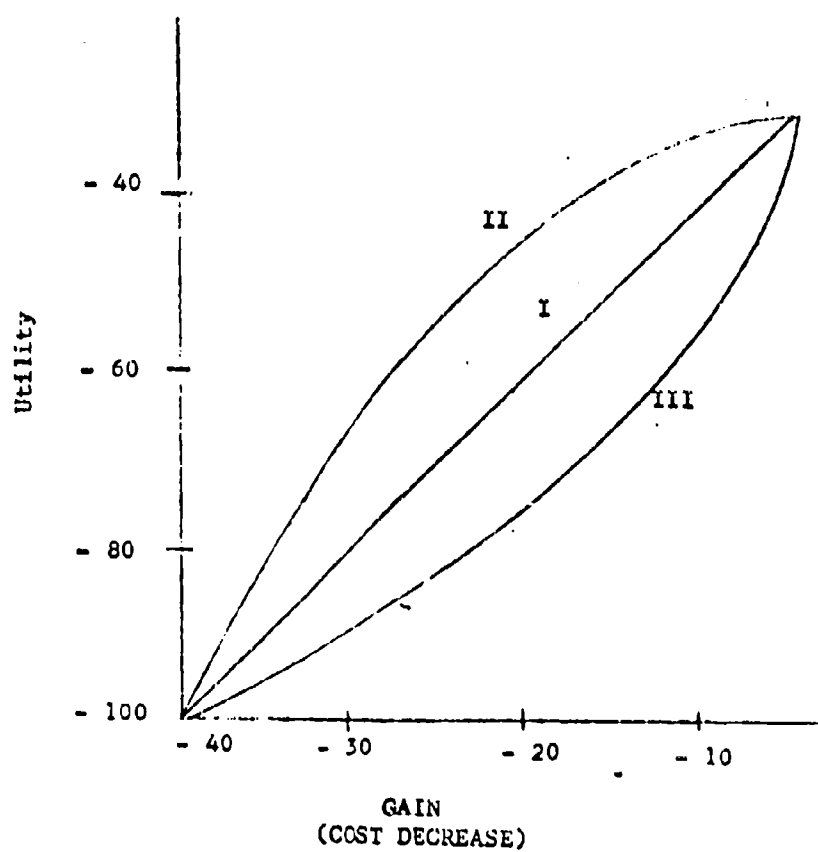


Figure 4.1

Three Possible Shapes of
Utility Functions

subjectively less valuable. Decision-maker II is "conservative" in the sense that in a risky situation he prefers the alternative with lower variability, even though both have the same expected monetary value. He is a risk averter.

Conversely, decision-maker III is a "risk taker" or "gambler" in the sense that he will pick the alternative with greater variability even though both have the same expected monetary value. His function, function III, reflects an increasing marginal utility of money, indicating that as dollar gains increase, additional gains become subjectively more valuable.

The von Neumann-Morgenstern utility theory has been used as a basis for experimentally determining the utility curves of individuals (for example, by Davidson, Suppes and Siegel (7) , by Halter and Dean (19) , by Mosteller and Nogel (96) and by Swalen (104)) and for experimentally determining the utility curves of firms (for example, by Green (79) and by Cramer and Smith (72)). As anticipated, the respondents preferred more expected gain to less, and preferred less variance.

The expected utility maxim. The expected utility maxim says that the individual should act as if

(1) he attaches numbers, called their utility to each possible outcome, and

(2) when faced with chance alternatives he selects the one with the greatest expected value of utility.

The opponents of the expected utility maxim have argued that the maxim is not the essence of rational behavior. They show instances in which human action differs from that dictated by the maxim. More pertinent, they show instances where reasonable action and the expected utility rule apparently contradict.

Allias has constructed several hypothetical decision situations (discussed by Savage (45 , pp. 101-103)) which seem to indicate that people do not always meet the conditions of rationality; that is, they can be trapped by certain question constructs. These opponents of the maxim claim that, while the axioms upon which modern utility theory is based have immediate appeal, they conceal objectionable assumptions.

The adherents of the expected utility maxim argue, to the contrary, that the existence of conflicts between actual behavior, the human is frequently confused and contradictory. As for the apparent contradictions between the expected utility maxim and reasonable behavior, adherents claim that opponents have misunderstood and misapplied the maxim.

The matter is still unsettled. The reader may find the pro and con discussions in the literature; see, for example (9,31,39,56).

The expected utility maxim, nevertheless, will be central to our subsequent development of an criterion function for our model. There are two reasons for this:

1. This writer believes that the arguments in favor of the expected utility maxim are quite convincing, especially for its application in areas where considerable uncertainty is prevalent,

such as a weapons acquisition program. The maxim has to be stretched, perhaps initially, to apply to the making of decisions in which surprise and the fun of gambling are important motivations. These, however, are not important objectives for a decision-maker in the allocation of large amounts of other people's money.

2. The expected utility maxim appears more consistent with observed behavior than the expected gain maxim. In most situations, decision-makers seek to avoid risks and are willing to forego gain in order to avoid them. Such behavior is incorporated within the expected utility maxim; while it is contrary to the expected gain maxim. Along with Keynes (27) we reject the contention that "an even chance of heaven or hell is precisely as much desired as the certain attainment of a rate of mediocrity," and support, instead, an apriori preference for the avoidance of risk.²⁰

Mathematical representation of the expected utility maxim.

In order to make the concept of expected utility operationally useful in a model, some functional relationship must be established.

²⁰ Risk aversion has not always been evident in our weapons programs, particularly, in regards to costs. However, because of recent "cost growth" problems and a reorientation of national priorities, budgets, and management emphasis, the Department of Defense (156, 158) placed renewed emphasis on the need to control costs and risk in our weapons programs. Cost and risk avoidance are now key characteristics of decision-making in our programs and as budgets shrink they should continue in that role.

If the utility function $U(C)^{21}$, is differentiable in the region of outcomes, C , under consideration, a Taylor's series expansion of the utility function about the expected outcome $E[C] = \mu$ yields

$$U(C) = U(\mu) + U'(\mu) \frac{(C - \mu)}{1!} + \frac{U''(\mu)}{2!} (C - \mu)^2 + \dots$$

Using the first three terms of the infinite series as an approximation to the utility function and assigning $U(\mu) = \mu$ (in accordance with the transformation properties of an interval scale, then

$$U(C) = \mu + U'(\mu)(C - \mu) + \frac{U''(\mu)}{2} (C - \mu)^2. \quad ^{22}$$

Taking expected values and recalling that $E[C] - \mu = 0$ and $E[(C - \mu)^2] = \sigma^2$, then

$$\begin{aligned} E[U(C)] &= \mu + U'(\mu)E[C - \mu] + \frac{U''(\mu)}{2} E[(C - \mu)^2] \\ &= \mu + \frac{U''(\mu)}{2} \sigma^2. \end{aligned}$$

The utility function for a risk averting-decision-maker, function II Figure 4.1, (which we assume our decision-maker to be)

²¹We will treat utility as a function of cost, C , since the military decision-maker is concerned with cost not profit in his decision-making.

²²Another mathematical representation of utility could be obtained by approximating the utility curve in Figure 4.1 with a quadratic curve. This type of approximation has been employed by Adelson (55) to solve a capital investment problem.

is characterized by diminishing marginal utility (curve is concave downward). Thus, $U''(\mu) < 0$ and the expected utility function can therefore be written in the form $E[U(C)] = \mu - R\sigma^2$ where $R = -\frac{U''(\mu)}{2} \geq 0$. The larger R is, the larger is the magnitude of the curvature of the utility function and therefore the greater the degree to which the curve reflects risk aversion. Thus, we call R the coefficient of risk aversion.

The expectation-variance expected utility function, $\mu - R\sigma^2$, is admittedly only an approximation to a true utility function.²³ It is therefore unable to exhibit the more subtle preference characteristics which some utility theorists (for example, Pratt (1960)) would ascribe to a true utility function. However, when high risk outcomes are being considered, the approximation does offer substantial progress toward a more refined basis of decision-making than can be obtained with the expected value criterion. Furthermore when no aversion toward risk is present, the function is consistent with the expected value criterion; that is, the function degenerates to the expected value criterion when $R = 0$.

²³ Variants of the expectation-variance function have been developed; for example Baumol (1960) proposed a criterion which substitutes $\mu - K\sigma$ for σ^2 in the function.

APPLICATION OF THE EXPECTED UTILITY
MAXIM TO THE RELIABILITY/
MAINTAINABILITY MODEL

In Chapter III we stated that the objective of our model was to minimize system life cycle cost

$$= \sum_{i=1}^N \sum_{j=1}^K LCC_{ij} x_{ij}.$$

In the previous subsection of this chapter, we developed an expectation-variance criterion function, maximize $\mu - R\sigma^2$, for use in decisions involving risk. Now we will combine these two results and develop the objective function used in our model.

In linear programming, maximizing any function subject to a set of restrictions is completely equivalent to minimizing the negative of the function subject to the same set of restrictions.²⁴ Thus, we can rewrite our expectation-variance criterion function as

$$\text{minimize } -\mu + R\sigma^2, \quad R \geq 0.$$

Furthermore since μ is equal to the expected cost, it is a negative value (cost being negative as shown in Figure 4.1). Therefore, $-\mu$ is a positive value and thus we may express our expectation-variance criterion function as

$$\text{minimize } \mu + R\sigma^2, \quad R \geq 0$$

where μ is the absolute value of the expected cost. In a utility

²⁴This follows immediately from the facts that (1) $f = -(-f)$, and (2) the larger f is, the smaller $-f$ is, so that (3) maximum $f = -$ minimum $(-f)$.

theory context, this expectation-variance criterion function, can be thought of as minimizing the disutility associated with negative gain (cost).

Our Chapter III criterion function expressed that we wished to minimize the sum of the subsystem life cycle cost estimates. We now relax the implied assumption of Chapter III, that the subsystem life cycle cost values are constants and let risk enter our model by treating the subsystem life cycle costs as random variables. With the subsystem life cycle cost values considered as random variables, it is no longer sufficient to say that we wish to minimize their sum. Instead, using our expected utility maxim, we state that our objective is to maximize the expected utility (minimize the disutility) of the system life cycle cost. Thus, our decision criterion function to be minimized is $E[U(C)] = \text{System Expected Life Cycle Cost} + R (\text{System Variance of Life Cycle Cost})$. To express this mathematically we will use the following property of a linear combination of independent variables: If x_1, x_2, \dots, x_n are independent random variables, a_1, a_2, \dots, a_n are constants, and

$$y = \sum_{i=1}^N a_i x_i,$$

then

$$E[y] = \sum_{i=1}^N a_i E[x_i]$$

and

$$\text{Var}[y] = \sum_{i=1}^N a_i^2 \text{Var}[x_i].$$

Using these relationships we can then write our decision criterion function as

$$\text{minimize } E[U(C)] = \sum_{i=1}^N \sum_{j=1}^K E[LCC_{ij}] x_{ij} + R \sum_{i=1}^N \sum_{j=1}^K \text{Var}[LCC_{ij}] x_{ij}$$

$$= \sum_{i=1}^N \sum_{j=1}^K [E[LCC_{ij}] + R \cdot \text{Var}[LCC_{ij}]] x_{ij}$$

for a system with N independent²⁵ subsystems with K reliability/maintainability alternatives for each subsystem.

where

$E[U(C)]$ = expected disutility of cost

$E[LCC_{ij}]$ = expected life cycle cost estimate for the j th reliability/maintainability alternative for the i th subsystem.²⁶

R = coefficient of risk aversion; $R \geq 0$.

²⁵As stated previously each subsystem is designed and developed by a separate design group. These groups work essentially independent of each other and the results they achieve in terms of cost and achievement of "design point" goals are independent in a statistical sense. Contingent and mutually - exclusive dependence frequently does exist and these types of dependence are considered later in this chapter.

²⁶The manner in which the expected value and variance estimates are obtained will be discussed in the last part of this chapter.

$\text{Var} [LCC_{ij}]$ = The variance estimate for the life cycle cost of the j th alternative for subsystem i ²⁷.

$x_{ij} = x_{ij}^2$ = 0 or 1 depending upon whether or not alternative j is selected for the i th subsystem.

Use of the Expectation - Variance Criterion Function

If R , the coefficient of risk aversion, were known or could be determined, the expectation-variance criterion-function could be used to select a particular combination of subsystem alternatives. The combination selected would be the optimum set in the sense that the objective function would be minimized. However, it seems pretty hard to evaluate the specific risk aversion that the decision-maker does or should have in a weapons acquisition program, and probably little can be added to what's already been said on this subject.

Therefore, our criterion function will not enable us to select a particular combination as being in any sense best but it will enable us to place alternative combinations into two categories: those that the decision-maker should consider and those that do not warrant further consideration because at least one other combination is clearly superior (that is, at least one other combination has the same or lower expected cost and a lower or the same variance). The set of combinations falling into the first category constitutes a schedule of efficient combinations each of which minimizes expected disutility for a particular level of risk aversion.

²⁷Ibid.

Consider a simple two-subsystem system example (Table 4.1) in which there are four reliability/maintainability alternatives per subsystem. Due to independence, the expected life cycle cost of a system is the sum of the expected life cycle costs of its subsystems and the variance of a system's life cycle cost is the sum of the variances of its subsystems. The possible systems (combinations) are enumerated in Table 4.2, and their expectation-variance combinations appear in Figure 4.2.

In accordance with the expectation-variance utility function, the decision-maker confronted with the problem of subsystem alternative selection would prefer systems (combinations) which have minimum variability for any given level of expected cost or minimum expected cost for any given level of variability. As shown in Figure 4.2, a particular system can conceivably be more risky than another system even though the expected cost of the first system is the same or greater than that of the second system. The first system is an inefficient system. Clearly some of the systems in Figure 4.2 are inefficient. For example, System 2, ($\mu_2 = 2255, \sigma_2^2 = 110$), has a larger expected cost than System 5, ($\mu_5 = 2215, \sigma_5^2 = 100$), and also a greater variability. The decision-maker responsible for system selection (that is, selecting the subsystem reliability/maintainability "design point" values) would want to avoid the inefficient systems (namely, systems 2, 3, 4, 6, 7, 8, and 14).

The model and solution technique to be developed provides a systematic method for reducing an unmanageably large number of

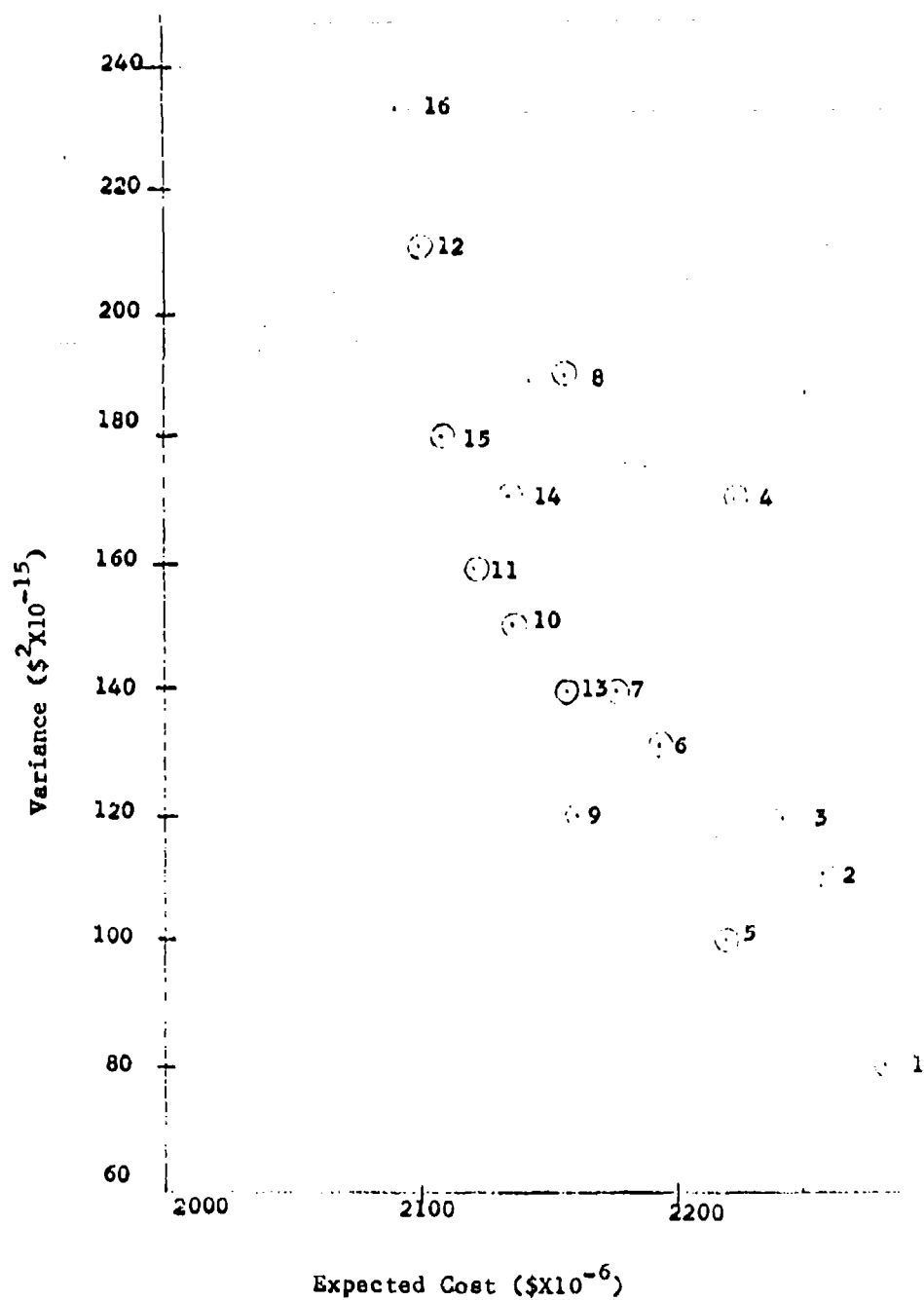
Table 4.1
Expected Cost and Variance Estimates
for Two Subsystems

Subsystem	Alternative	Expected Cost (\$ $\times 10^{-6}$)	Variance (\$ $\times 10^{-12}$)
1	1	1180	40,000
1	2	1115	60,000
1	3	1060	80,000
1	4	1050	100,000
2	1	1100	40,000
2	2	1075	70,000
2	3	1060	80,000
2	4	1040	130,000

Table 4.2

System Expected Costs and Variances
For Two-Subsystem Example

System	Subsystem Alternatives In System		Expected Cost (\$X10 ⁻⁶)	Variance (\$X10 ⁻¹²)
	S.S. 1	S.S. 2		
1	1	1	2280	80,000
2	1	2	2255	110,000
3	1	3	2240	120,000
4	1	4	2220	170,000
5	2	1	2215	100,000
6	2	2	2190	130,000
7	2	3	2175	140,000
8	2	4	2155	190,000
9	3	1	2160	120,000
10	3	2	2135	150,000
11	3	3	2120	160,000
12	3	4	2100	210,000
13	4	1	2150	140,000
14	4	2	2125	170,000
15	4	3	2110	180,000
16	4	4	2090	230,000



Expected Cost ($\$ \times 10^{-6}$)

Figure 4.2

Expectation-Variance Combinations for Two-Subsystem Example

expectation-variance combinations (K^N alternative combinations for N subsystems with K alternatives each) to a small, tractable group of feasible (that is, they satisfy the external and internal constraints), efficient systems, each of which minimizes disutility for a specific level of risk aversion. The group generated is the schedule of efficient systems (combinations).

THE SYSTEM CONSTRAINTS UNDER UNCERTAINTY

Consider now the system external constraint (that is, constraint 1-5 pp. 52-53) and let the reliability, maintainability, cost, and weight values associated with the subsystem alternatives be considered as random variables. Then each external constraint becomes probabilistic in nature and can be expressed in the form of a "chance-constraint"; that is, each constraint is of the form,

$$\Pr \left[\sum_{i=1}^N \sum_{j=1}^K a_{ij} x_{ij} \leq \text{or} \geq b \right] \geq \beta_h,$$

where "Pr" means "probability," the a_{ij} are random variables, b is the stipulated system level for the h th factor, and if $0 \leq \beta \leq 1$, the constraint means that it is permissible to violate this constraint with at most, probability $(1 - \beta)$ in any admissible choice of x_{ij} values.

Chance-constrained programming was formulated by Charnes, Cooper, and Symonds (68) and Charnes and Cooper (65) , and has since

been further developed and applied by Charnes and Cooper (66, 67), Charnes, Cooper and Thompson (69), Hillier (84), Kataoka (87), Kirby (134), Miller and Wagner (94), Naslund (97), Naslund and Winston (98), Sinhal (16), and Van DePanne and Popp (107).

Deterministic Equivalent Forms

The first step in solving a programming problem in which there are chance-constraints is to reduce the chance-constraints to their deterministic equivalent forms. Consider a typical constraint,

$$\Pr \left[\sum_{i=1}^N \sum_{j=1}^K a_{ij} x_{ij} \leq b_h \right] \geq \beta_h$$

Assume that the a_{ij} are statistically independent²⁸ and estimates of the expected values and variances of the a_{ij} have been obtained²⁹ and b_h and β_h are stipulated constants. Denote the expected values and the variances by $E[a_{ij}]$ and $\text{Var}[a_{ij}]$, respectively. Then the chance-constraint might be stated as follows: restrict

$$\mu = \sum_{i=1}^N \sum_{j=1}^K E[a_{ij}] x_{ij}$$

²⁸As stated previously, achievement of subsystem goals can be considered statistically independent.

²⁹The manner in which the expected value and variance estimates are obtained will be discussed in the last part of this chapter.

and

$$\sigma^2 = \sum_{i=1}^N \sum_{j=1}^K \text{Var} [a_{ij} x_{ij}] \quad 30$$

in such a way that the probability of

$$\sum_{i=1}^N \sum_{j=1}^K a_{ij} x_{ij}$$

exceeding b_h is less than $\alpha_h = (1 - \beta_h)$. The restriction which satisfies the stipulated probability

$$\text{Pr} \left[\sum_{i=1}^N \sum_{j=1}^K a_{ij} x_{ij} \geq b_h \right] \leq \alpha_h$$

is given by the constraint ³¹

$$\frac{b_h - \mu}{\sigma} \geq z_h$$

where z_h is the standardized random variable corresponding to the stipulated probability α_h . If the distribution of

$$\sum_{i=1}^N \sum_{j=1}^K a_{ij} x_{ij}$$

³⁰ Actually, $\sigma^2 = \sum_{i=1}^N \sum_{j=1}^K \text{Var} [a_{ij} x_{ij}^2]$, but since $x_{ij} = 0$ or 1 , $x_{ij}^2 = x_{ij}$. Therefore, we can replace x_{ij}^2 with x_{ij} in our expression for σ^2 .

³¹ A detailed expository treatment on developing deterministic equivalents for chance-constraints may be found in Charnes and Cooper (66), Kataoka (87), and Hillier and Lieberman (23).

is normal, and

$$y = \frac{\sum_{i=1}^N \sum_{j=1}^K a_{ij} x_{ij} - \mu}{\sigma}$$

then the correspondance between Z_h (which for normal distribution is called the standard normal deviate) and α_h is

$$\alpha_h = \int_{Z_h}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy.$$

Values of Z_h for stipulated values of α_h can be found in standard normal tables available in basic statistics books; for example, if $\alpha_h = 0.10$, then $Z_h = 1.28$ is found in such a table.

Of course, the actual underlying distribution of the a_{ij} are in most cases unknown. Therefore, the distribution of their sum is also unknown. However, since the a_{ij} are independent random variables, with finite means and variances, which are either identically distributed or uniformly bounded, then (by the Lindeberg Theorem) the Central Limit Theorem will hold and the sum of these random variables will be approximately normal, if N is large. It is difficult to give a specific N beyond which the Central Limit Theorem applies, and approximate normality can be assumed. This of course, does depend upon the form of the underlying a_{ij} distributions. In our model, since the a_{ij} random variables themselves are made up of component distributions, they themselves should tend towards normality. Thus, moderate N sizes should be sufficient.

Further details on the Central Limit Theorem may be found in any book on advanced probability theory, e.g., Feller (11), Fraser (14), and Loeve (30). Also, an excellent discussion of the theorem and its use in evaluating risky investments may be found in Hillier (22, pp. 24-29).

Rearranging the constraint we can write

$$\mu + z_h \sigma \leq b_h$$

or

$$\sum_{i=1}^N \sum_{j=1}^K E[a_{ij}] x_{ij} + z_h \sqrt{\sum_{i=1}^N \sum_{j=1}^K \text{Var}[a_{ij}] x_{ij}} \leq b_h$$

as the deterministic equivalent for a chance-constraint of the form,

$$\Pr \left[\sum_{i=1}^N \sum_{j=1}^K a_{ij} x_{ij} \leq b_h \right] \geq \beta_h$$

where a_{ij} are random variables and b_h and β_h are stipulated constants.

In a similar manner it can be shown that a chance-constraint of the form,

$$\Pr \left[\sum_{i=1}^N \sum_{j=1}^K a_{ij} x_{ij} \geq b_h \right] \geq \beta_h.$$

has as a deterministic equivalent

$$\sum_{i=1}^N \sum_{j=1}^K E[a_{ij}] x_{ij} - z_h \sqrt{\sum_{i=1}^N \sum_{j=1}^K \text{Var}[a_{ij}] x_{ij}} \geq b_h$$

when a_{ij} are independent random variables, b_h and β_h are stipulated constants, and Z_h is the standard normal deviate as defined on page 80 .

Linearization of the deterministic equivalent forms. The problem now is to reduce these deterministic equivalent forms further to a more tractable form. The objective will be to linearize the constraints so that an integer linear programming algorithm can be used to solve the model. The basic approach is suggested by the following obvious result.

Fundamental Lemma: assume that $g_1(x) \leq g_2(x)$ for all admissible x . Consider a solution x which is feasible if and only if $g_1(x) \leq k$. (that is, x satisfies all other conditions for feasibility). If $g_2(x) \leq k$, then x is feasible. Thus, if $g_1(x) \leq k$ represents some exact deterministic equivalent form of the constraint, $g_2(x) \leq k$ will represent a linear constraint that is uniformly tighter.

Our nonlinear term in the deterministic equivalent constraints is of the form

$$\sqrt{\sum_{k=1}^M \sigma_k^2 x_k}.$$

$$\text{where } \sum_{k=1}^M \text{ replaces } \sum_{i=1}^N \sum_{j=1}^K, \quad \sigma_k^2 = \text{Var}[a_{ij}],$$

$$\text{and } x_k = x_{ij}.$$

It can be shown that since x_k are restricted to values of zero or one,

$$\sqrt{\sum_{k=1}^M \sigma_k^2 x_k} \leq f(x)$$

where

$$f(x) = \sigma - \sum_{k=1}^M (1 - x_k) \left[\sigma - \sqrt{\sigma^2 - \sigma_k^2} \right],$$

where

$$\sigma^2 = \sum_{k=1}^M \sigma_k^2.$$

The proof utilizes the following condition:

$$\begin{aligned} \sum_{k=1}^M \left[\sqrt{\sigma^2 - \sum_{h=1}^{k-1} \sigma_h^2 (1 - x_h)} - \sqrt{\sigma^2 - \sum_{h=1}^k \sigma_h^2 (1 - x_h)} \right] \\ \geq \sum_{k=1}^M \left[\sigma - \sqrt{\sigma^2 - \sigma_k^2 (1 - x_k)} \right]. \end{aligned}$$

This inequality can be seen by referring to Figure 4.3. If we take $B - A = B' - A'$, the inequality says $\sqrt{B} - \sqrt{A} \geq \sqrt{B'} - \sqrt{A'}$, which follows intuitively from the figure, since $D - E \geq D' - E'$.

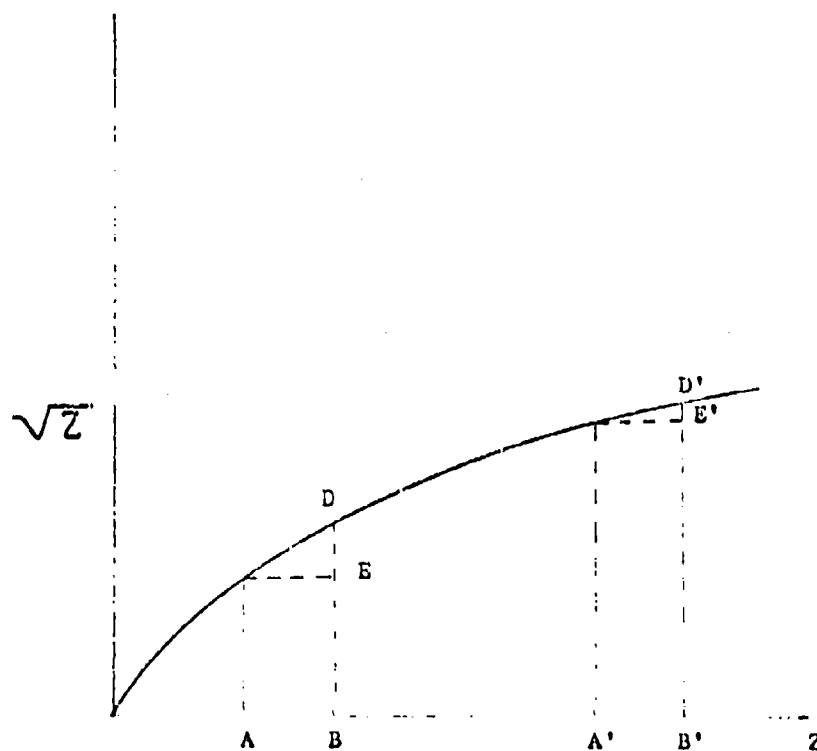


Figure 4.3

Illustration of Inequality Used in Proof

We then use the following equality:

$$\sqrt{\sum_{k=1}^M \sigma_k^2 x_k} = \sigma - \sum_{k=1}^M (1 - x_k) \quad X$$

$$\left\{ \sqrt{\sigma^2 - \sum_{h=1}^{k-1} \sigma_h^2 (1 - x_h)} = \sqrt{\sigma^2 - \sum_{h=1}^k \sigma_h^2 (1 - x_h)} \right\}$$

Combining this equality with the previously stated inequality gives the desired inequality. Figure 4.4 gives a picture of this linear approximation, $f(X)$. The curve indicates the square-root function and line I indicates $f(X)$. The two functions are equal at

$$\sigma = \sqrt{\sum_{k=1}^M \sigma_k^2}.$$

If the solution is further restricted (as our solution is) so that

$$\sum_{i=1}^N \sum_{j=1}^K x_{ij} = N < NK - 1,$$

it also follows that

$$\sqrt{\sum_{k=1}^M \sigma_k^2 x_k} \leq f(X) - C,$$

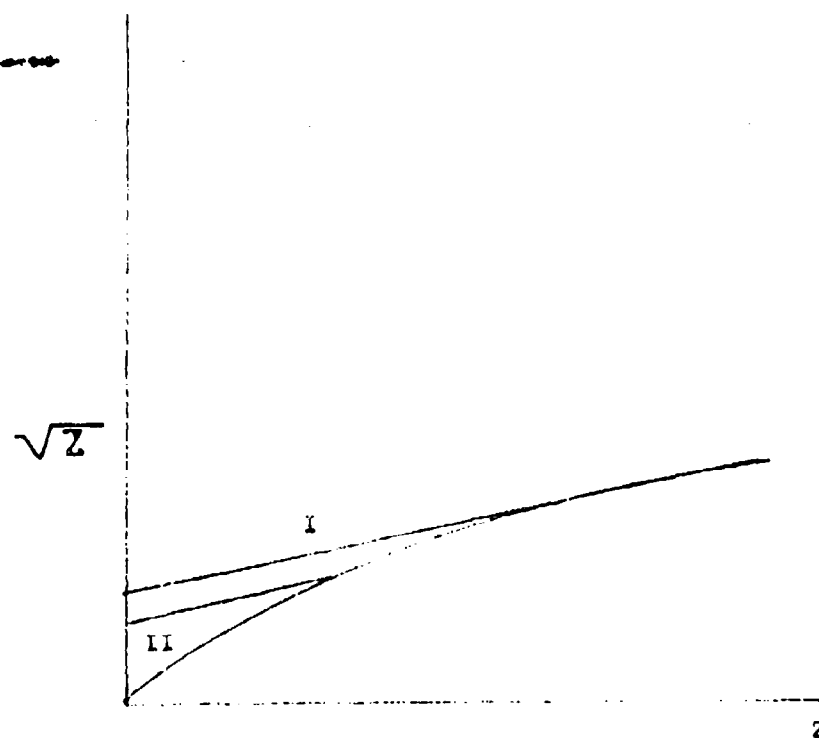


Figure 4.4

Illustration of Linear Approximation

where

$$C = \sigma - \sum_{k \in K} \left[\sigma - \sqrt{\sigma^2 - \sigma_k^2} \right] - \sqrt{\sigma^2 - \sum_{k \in K} \sigma_k^2},$$

where the set K contains the values of k corresponding to the $(NK - N)$ smallest values of σ_k^2 ($k = 1, \dots, M$). Referring again to Figure 4.3, line II indicates $f(X) - C$. Line II is equal to the curve at the point equal to the sum of N largest values of σ_k^2 ($k = 1, \dots, M$). This adjusted linear function, $f(X) - C$, provides an improved approximation and it is the one incorporated in the present model.

The form of a deterministic equivalent constraint incorporating $f(X) - C$ for $\sqrt{\sum_i \sum_j \text{Var}[a_{ij}] x_{ij}}$ would be

$$\sum_{i=1}^N \sum_{j=1}^K E[a_{ij}] x_{ij} + z_h [f(X) - C] \leq b_h$$

where $f(X)$ is as defined on page 91 and C is as defined above. Since $f(X) - C$ is always greater than or equal to the square-root function it replaced, its effect is to provide a more conservative solution, in that, the chance-constraints are satisfied at a probability level somewhat greater than that stipulated by β_h .

SUMMARY OF THE RELIABILITY/MAINTAINABILITY MODEL UNDER UNCERTAINTY

Before concluding this section, let us summarize our development up to this point. Basically speaking, the present approach to decision-making under uncertainty relies on only two substantive proposals:

- (1) The decision-maker is a risk averter; that is, his utility of money function is positively sloped and concave downward, and
- (2) His decision criteria is the maximization of expected utility (in our case, since we are dealing with cost, we use the minimization of expected disutility as his criterion function).

On the basis of these proposals, we developed an operational model that is expressed as follows:

$$\min E[U(C)] = \sum_{i=1}^N \sum_{j=1}^K \left[E[LCC_{ij}] + R \cdot \text{Var}[LCC_{ij}] \right] x_{ij}$$

subject to some or all of the following constraint:

$$1) \quad \Pr \left[\sum_{i=1}^N \ln \sum_{j=1}^K \left[1 - \frac{\lambda_{ij} t_o \cdot \text{MTTR}_{ij}}{t_o + t_{no}} \right] x_{ij} \geq \ln A \right] \geq \beta_A$$

$$2) \quad \Pr \left[\sum_{i=1}^N \sum_{j=1}^K w_{ij} x_{ij} \leq w \right] \geq \beta_w$$

$$3) \quad \Pr \left[\sum_{i=1}^N \sum_{j=1}^K R\&D_{ij} x_{ij} \leq R\&D \right] \geq \beta_{R\&D}$$

$$4) \quad \Pr \left[\sum_{i=1}^N \sum_{j=1}^K \lambda_{ij} x_{ij} \leq \lambda \right] \geq \beta_\lambda$$

$$5) \quad \Pr \left[\sum_{i=1}^N \sum_{j=1}^K (MTTR_{ij} - MTTR) \lambda_{ij} x_{ij} \leq 0 \right] \geq \beta_{MTTR}$$

$$6a) \quad -N + \sum_{i=1}^N \sum_{j=1}^K x_{ij} \geq 0$$

$$6b) \quad 1 - \sum_{j=1}^K x_{ij} \geq 0, \quad i=1, \dots, N$$

$$x_{ij} = 0 \text{ or } 1,$$

where λ_{ij} , $MTTR_{ij}$, w_{ij} , and $R\&D_{ij}$ are random variables; A , w , $R\&D$, λ and $MTTR$ are desired system levels; β_A , β_w , $\beta_{R\&D}$, β_λ , and β_{MTTR} are stipulated probability levels; and N is the number of subsystems and K is the number of alternatives per subsystem.

To solve the model, the chance-constraints are replaced by their linearized deterministic equivalents which incorporate uniformly tighter linear functions for their nonlinear terms. Thus, the model provides a more conservative solution than that required by the stipulated β probability levels. The major assumptions of the model are that subsystem alternative values are statistically independent and that system values, which are equal to the sum of the values for the selected subsystem alternatives, are approximately normal.

To conclude development of the model we will now discuss contingent and mutually-exclusive dependence. Then we will conclude this chapter with a discussion on obtaining the required probabilistic inputs for the model.

CONTINGENT AND MUTUALLY- EXCLUSIVE ALTERNATIVES

Two types of dependence which commonly exist are the result of contingent and mutually-exclusive alternatives. Alternatives i and j are contingent, if selection of alternative j , requires that one also select alternative i ; however, i may be undertaken without j . To represent this situation, all we need to do is add a constraint of the form

$$y_i - y_j \geq 0$$

It is also possible that alternatives i and j are mutually contingent; that is, either both must be selected or rejected. To represent

this situation, all we need to do is add a constraint of the form

$$y_i - y_j = 0.$$

If three alternatives, say, i , j , and k were mutually contingent, all we need to do is add a constraint of the form

$$2y_i - y_j - y_k = 0.$$

As another possibility, suppose that if one of the alternatives, i , j , or k , is selected, then one must not select either of the remaining two alternatives. To represent this situation, all we need to do is add a constraint of the form

$$1 - x_i - x_j - x_k \geq 0.$$

In our model we use such mutually-exclusive constraints to insure that no more than one alternative is selected for each subsystem.

Finally suppose that at most any two of the three alternatives, i , j , and k , can be selected. This can be represented by the constraint

$$2 - x_i - x_j - x_k \geq 0$$

Thus, we see that the mathematical programming formulation provides considerably versatility in handling these types of dependence. In most cases, only a single constraint need be added to the problem to cover the particular contingent or mutually-exclusive dependence desired. In Chapter VI, we will consider both mutually-contingent and mutually-exclusive alternatives in the example problems we solve.

OBTAINING MODEL INPUT ESTIMATES

Probabilistic Estimates

The actual underlying distribution of the reliability, maintainability, weight, and cost random variables are unknown and therefore unavailable for use as a basis for the selection decision. The decision, however, must be made and when made it should utilize as fully as possible whatever information is available, even imperfect information which may be obtainable only in the form of judgmental estimates. In our problem the engineering estimates would be such judgmental estimates. Resorting to such information is not a matter of choice but is a necessity, particularly in the conceptual design stages, in a weapon's acquisition program.

As discussed in Chapter III, the basic source of input data required for the model are the proposals describing the reliability/maintainability programs needed to achieve different subsystem reliability/maintainability levels. These proposals would contain engineering estimates for the normal, high, and ultra high reliability/maintainability input parameters including R&D costs and weight estimates. These engineering estimates would then be used in a cost model to determine the other required life cycle cost elements. In Chapter III we treated these estimates as point estimates of system parameters. Now, we will expand our view and see how we might

obtain probabilistic rather than point estimates.

There are several probability distributions that may be discussed relative to their suitability to describe subjective probabilistic estimates; the uniform distribution, the normal distribution, and the beta distribution appear to be the ones most frequently mentioned in the literature. In this study we do not recommend any distribution as being best. The model can use inputs in terms of any one or a combination of these distributions. We require mean and variance estimates and are not directly concerned with the exact form of the distribution used to convert the engineering estimates into our required mean and variance parameters.

Uniform method of estimation. Use of the uniform distribution for approximately describing the subsystem random variables would require that two estimates rather than a single point estimate be supplied. These estimates would be in the form of endpoints of a range within which the estimator believed the value would lie. The assumption is then made that the mean value lies mid-way between the two estimates provided and the variance is computed from $\frac{(b - a)^2}{12}$, where b and a are the two endpoint estimates. The main advantage in using a uniform distribution as a prior is that it requires the least amount of information to give a probabilistic estimate. Its main disadvantage is that

it does not permit the estimator to give the most likely value of his estimate. In most cases our prior distribution is really not flat. A unimodal prior distribution with peak over what we believe to be the most likely value seems more appropriate.

Normality method of estimation. Use of the normal distribution for approximately describing certain random variable estimates is in some cases quite reasonable. Even though the range of the normal distribution is from $-\infty$ to $+\infty$, the contention is often made that the probabilities of values occurring unrealistically far from the mean are very small and such values may not substantially detract from the reasonableness of the approximation. Also when a large number of component elements, many of which may be independently affected by numerous diverse factors, are known to cumulatively determine the subsystem values, then by virtue of the Central Limit Theorem the distribution of the overall subsystem values may be expected to be approximately normal.

Under assumptions of normality, the subsystem design group estimator could supply his estimate of the expected subsystem values by asking himself "What is an estimated value such that I would be as ready to bet that the actual value will be above this estimate as I would be to bet that the actual value will be below this estimate?" To obtain the estimate of the variance of the distribution, the estimator could ask himself "For what (symmetric) range around the expected value is the actual value as likely to be

within this range as outside this range?" Suppose that the estimator decides there is a 50-50 chance that the actual mean time to repair will be within one hour of the expected mean time to repair which he estimated as three hours. That is, he judges there is a 50-50 chance that the actual mean time to repair will be between two and four hours, and a 50-50 chance that the actual cost will be somewhere outside the two-four hour range. This narrative description of the variability can be used to estimate the variance of the maintainability distribution by noting that the probability is .50 a normally distributed random variable assumes a value within $\pm .67$ standard deviations of its mean value. The major disadvantage in using a normal distribution is its symmetry. The estimator may believe that his uncertainty is skewed left or skewed right and such information is removed when a normal distribution is assumed. For example, many cost factors have a realistic minimum but no obvious maximum suggesting a distribution that is skewed right. On the other hand, some inputs often tend to have uncertainties skewed left.

PERT method of estimation. Another method which finds rather wide acceptability for describing certain random variables is that method employed in PERT. The distribution commonly used to describe the random variable is the beta distribution, which has the desirable property of being contained in a finite interval and which can be skew or symmetric depending on the location of the mode, m , relative to the endpoints, a and b , of the interval.

Under the assumptions of the PERT method, three judgments are required of the estimator (that is, an optimistic estimate a , a most likely estimate m , and a pessimistic estimate b). These three judgments can be used to approximate the expected value by $\frac{a + 4m + b}{6}$ and variance by $\frac{(b - a)^2}{6}$.

It is worth noting that any sensible intuitive solution of the reliability/maintainability conceptual design problem would somehow (1) use past experience, engineering judgment, etc., to provide some idea of the attainable subsystem reliability/maintainability levels and the effort needed for achieving these levels, and (2) tend to allocate effort where the results to be attained would be "best" according to some form of criteria. All we have done in formulating our model and developing a solution technique for it is to provide a method to perform these very same functions but in a formal quantitative fashion.

Present Value Adjustment of Cost Estimates

The life cycle

cost element estimates associated with each reliability/maintainability alternative extend over multiple time periods (for example, the anticipated life cycle of a weapon system may be twenty years). Therefore, the estimates of expected values and variances of the costs associated with each alternative must be adjusted to present values for the model. Hence all expected values and standard deviations in the i th period are adjusted by the term $\frac{1}{(1 + r)^i}$

where r is the discount factor (a value of 10 percent is currently in use by the military) which reflects the time value of money.

All subsequent examples will assume this adjustment has been made.

CHAPTER V

MODEL SOLUTION TECHNIQUE

In this chapter we present the solution technique developed for solving the reliability/maintainability conceptual design model. First, the search procedure used for generating the schedule of efficient systems is discussed. Next we discuss the computer program used for solving the model. Computer input requirements and computer outputs are explained and the program itself is described.

SEARCH PROCEDURE

Consider the set of feasible and non-feasible expectations-variance combinations in Figure 5.1. The expected disutility of any point (μ, σ^2) in the set is $E[U(C)] = \mu + R\sigma^2$, and the direction of decreasing expected disutility is in the direction of decreasing expected costs and decreasing variance. Geometrically, minimization of the expected disutility function involves passing the expected disutility indifference curve, $\sigma^2 = -1/R\mu + 1/R E[U(C)]$ through the set of expectation-variance combinations until that feasible, efficient system is found for which the expected disutility is a minimum. (In actuality we find our minimum expected disutility system mathematically by means of our computer programmed mathematical

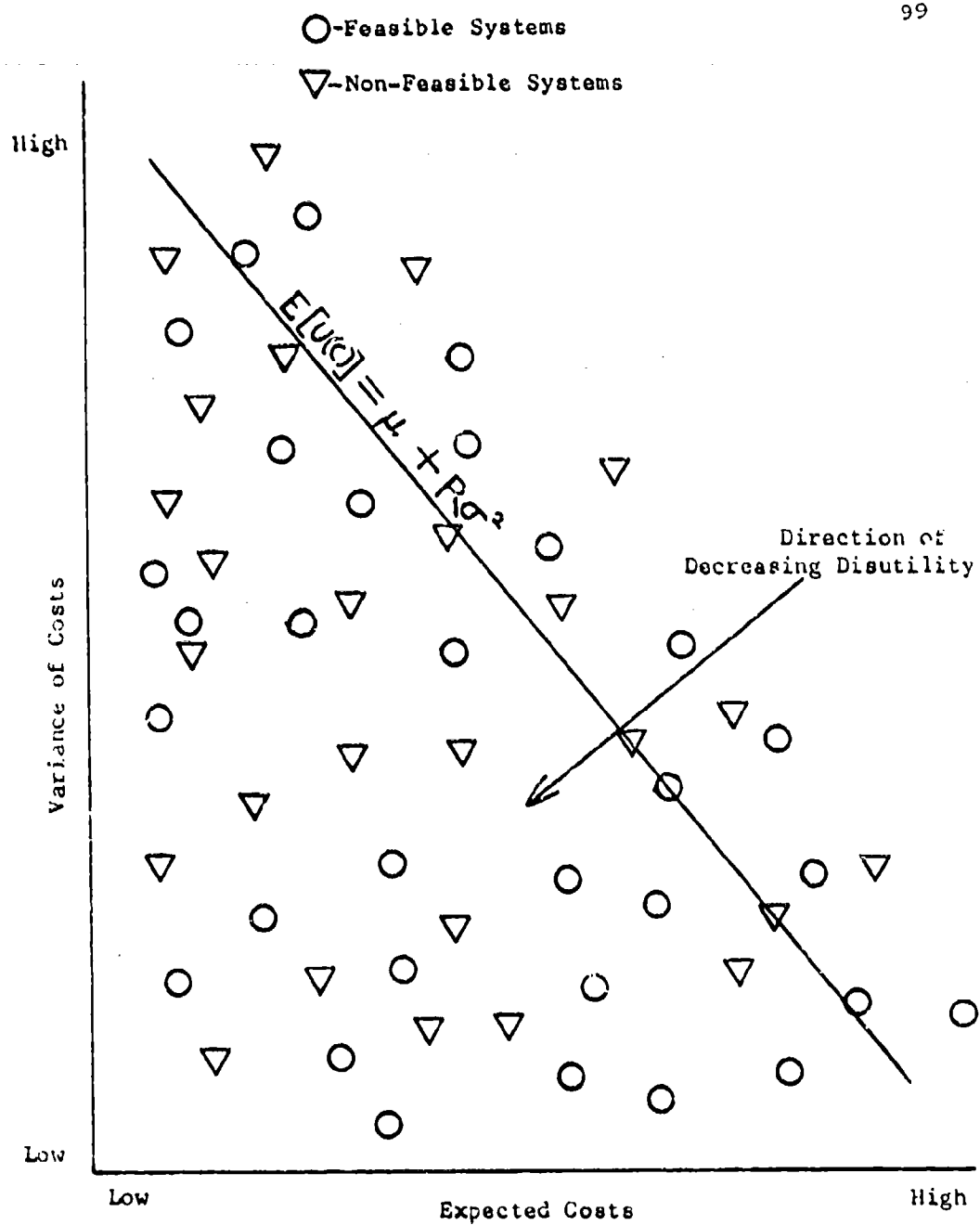


Figure 5.1

Direction of Decreasing Expected Disutility

programming algorithm.) Such a feasible, efficient, minimum expected disutility system will be denoted as a FEMD system. The question naturally arises as to how the ability or willingness of the decision-maker to tolerate risk is transformed into a specific number, the coefficient of risk aversion R . As previously stated, we do not expect the decision-maker to be able to specify such a specific number. Thus, the approach will be to generate the set of all FEMD systems. The systems in this set comprise the schedule of efficient systems. The decision-maker can then evaluate the relative merits of the systems in this greatly reduced set, being assured that each system in the set is feasible and efficient. Confronted with this reduced set he can employ whatever criterion of choice he deems appropriate.

Note that the slope of the expected disutility indifference curve is the negative of the coefficient of risk aversion. By determining the FEMD systems corresponding to various levels of risk aversion (that is, by passing the expected disutility indifference curve through the feasible expectation-variance combinations at various slopes), a set of such systems is generated as in Figure 5.2.

Observe in Figure 5.3 that if the decision-maker has no aversion toward risk (that is, $R = 0$), the optimal system would be that FEMD system which minimizes expected costs. On the other hand, if the decision-maker is extremely averse toward risk (that is, $R \rightarrow \infty$), the optimal system would be that FEMD system which minimizes variance. Each of the other FEMD systems would be optimal for certain

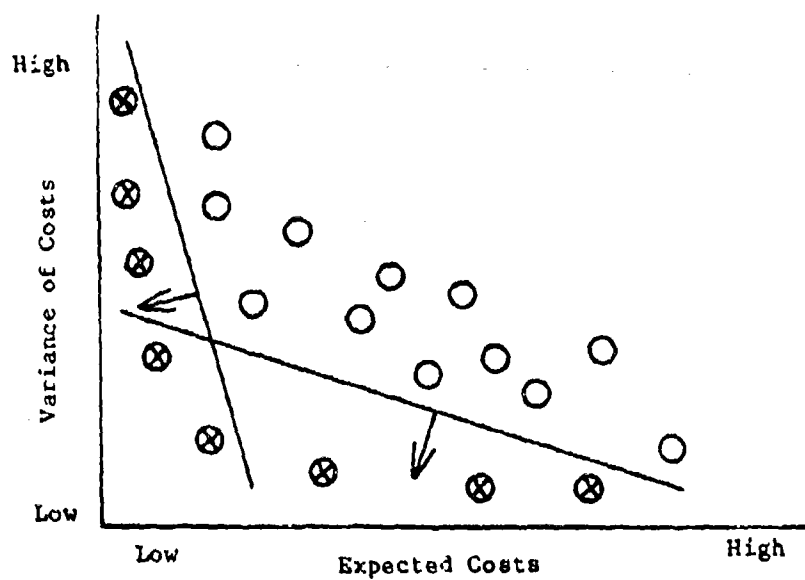


Figure 5.2

FEMD Systems Obtained from Feasible
Expectation-Variance Combinations

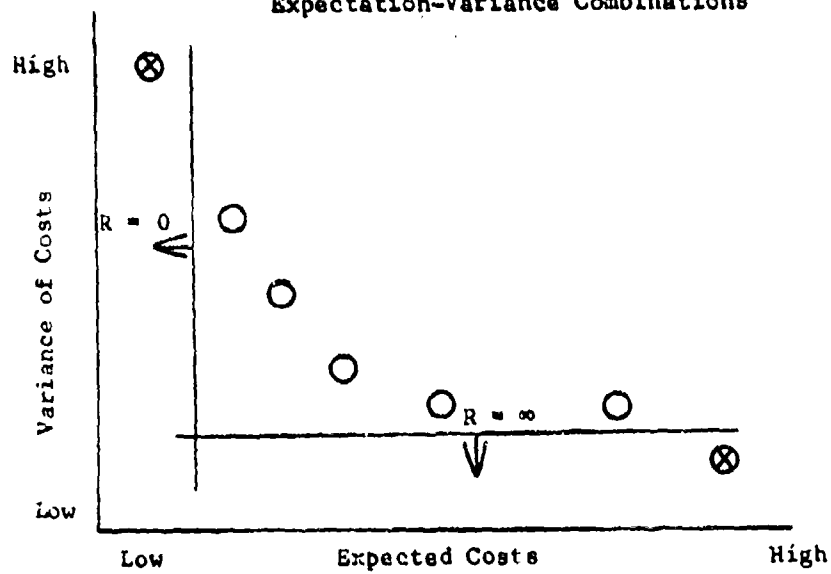


Figure 5.3

Minimum Expected Cost FEMD System
And Minimum Variance FEMD System

corresponding ranges of risk aversion coefficients.

Skipping FEMD Systems

As indicated in Figure 5.4, letting $R = R_2$ and passing the expected disutility indifference curve, $\sigma^2 = -1/R_2 \mu + 1/R_2 E[U(C)]$, through the feasible set of systems will produce the FEMD system (μ_2, σ_2^2) . If the next trial of R is R_1 , FEMD system (μ_1, σ_1^2) will be produced and system (μ_x, σ_x^2) will have been skipped.

The nature of skipped systems and the selection of R 's may become more meaningful by further considering the two-subsystem system example presented in Chapter IV. Assume certain constraints have been imposed which preclude some of the combinations (Table 4.2) from being in the feasible set. Namely, assume systems 2, 3, 6, and 7 are non-feasible. The remaining feasible combinations are presented in Figure 5.5. Suppose two trial values of R are chosen (0.0 and 1.0) and the two indifference curves (at slopes $-1/R$ for each R) are passed through the set of feasible systems. By so doing, systems 16 and 1 are found to be the FEMD systems for R values of 0.0 and 1.0 respectively.

The question arises as how one might check for skipped FEMD systems corresponding to untried values of R in the range $0.0 < R < 1.0$ (and indeed some have been skipped; namely FEMD systems 15, 11, 9, and 5). Certainly it would be desirable to have some systematic procedure for obtaining additional FEMD systems, if they exist, without testing an exorbitantly large number of R 's.

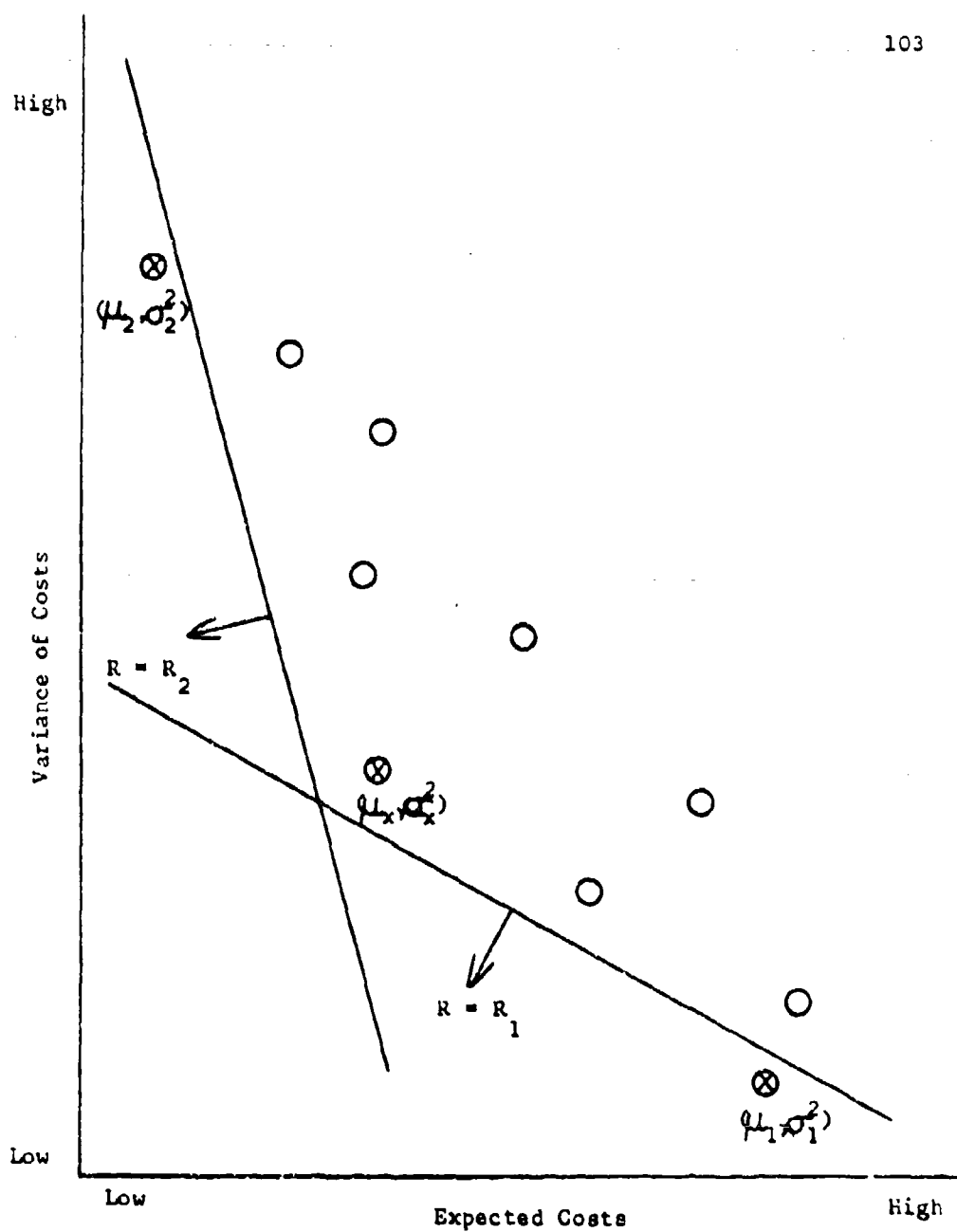


Figure 5.4

Skipping A FEMD System

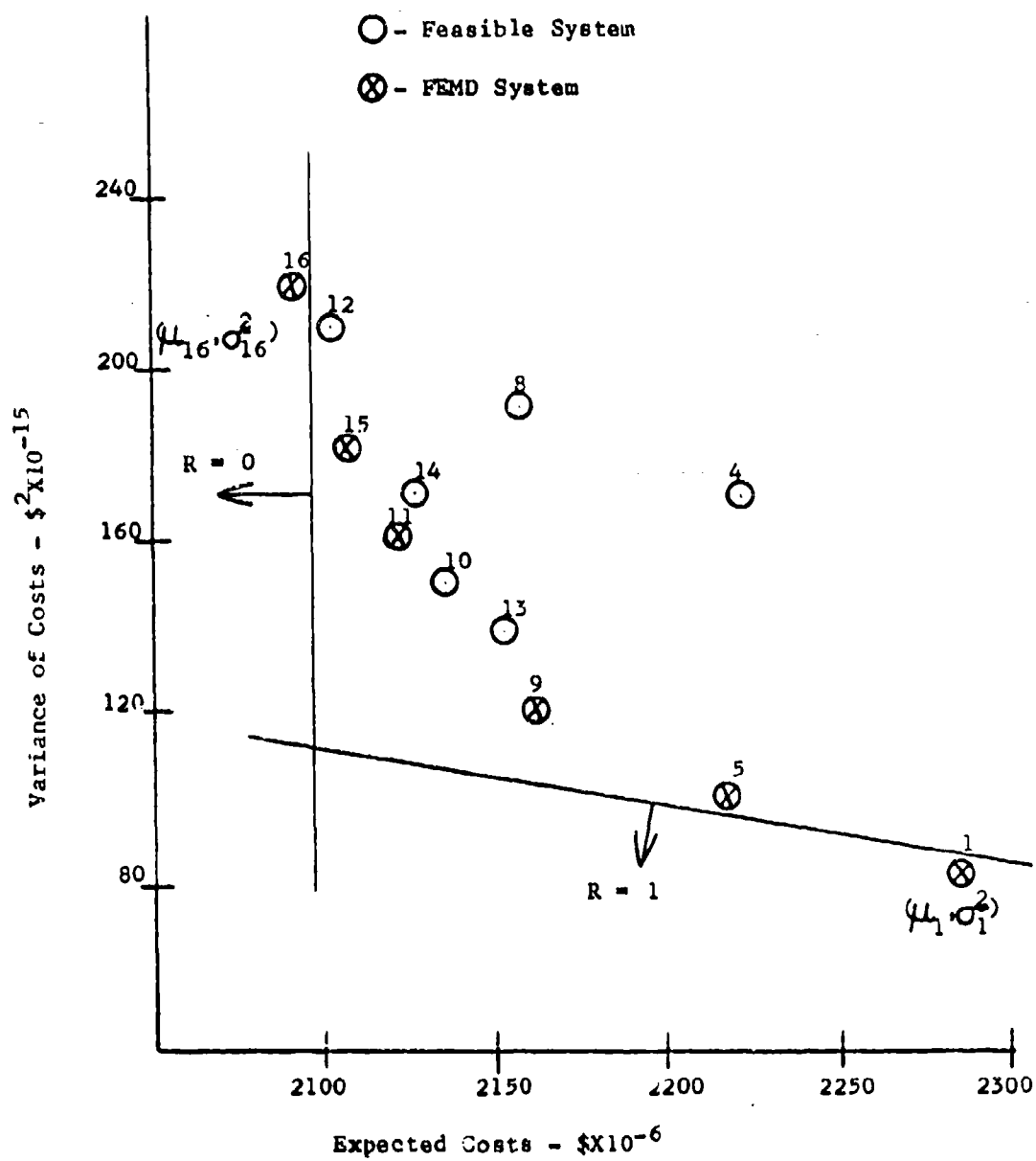


Figure 5.5

Feasible and FEMD Systems for Two-Subsystem Example

Testing between R's which have different FEMD systems. When

a change from one value of R to another value of R results in generation of a different FEMD system, one or more FEMD systems corresponding to intermediate values of R, might have been skipped. In the two-subsystem example, the range $0.0 < R < 1.0$, exemplifies this case. As indicated in Figure 5.6, when the two FEMD systems (μ_1, σ_1^2) and (μ_2, σ_2^2) are known to exist, a testing procedure is desired which will either (1) determine any additional FEMD systems in the enclosed region which have been skipped or (2) indicate that no additional FEMD systems exist in the region.

If such an unknown FEMD system exists, there must be some value of R for which the unknown FEMD system has less disutility than either of the two known FEMD systems. This unknown FEMD system will be revealed by trying that value of R which causes the indifference curve to pass through both known expectation-variance combinations simultaneously. Since this value of R occurs when $\mu_1 + R \sigma_1^2 = \mu_2 + R \sigma_2^2$, select as a trial value,

$$R = \frac{\mu_2 - \mu_1}{\sigma_1^2 - \sigma_2^2}$$

If such a system does not exist, both (μ_1, σ_1^2) and (μ_2, σ_2^2) are FEMD systems for this new R value, since they both have equal (and minimum) expected disutility for that value of R. The computational scheme actually used for generating FEMD systems, however, will indicate either (μ_1, σ_1^2) or (μ_2, σ_2^2)

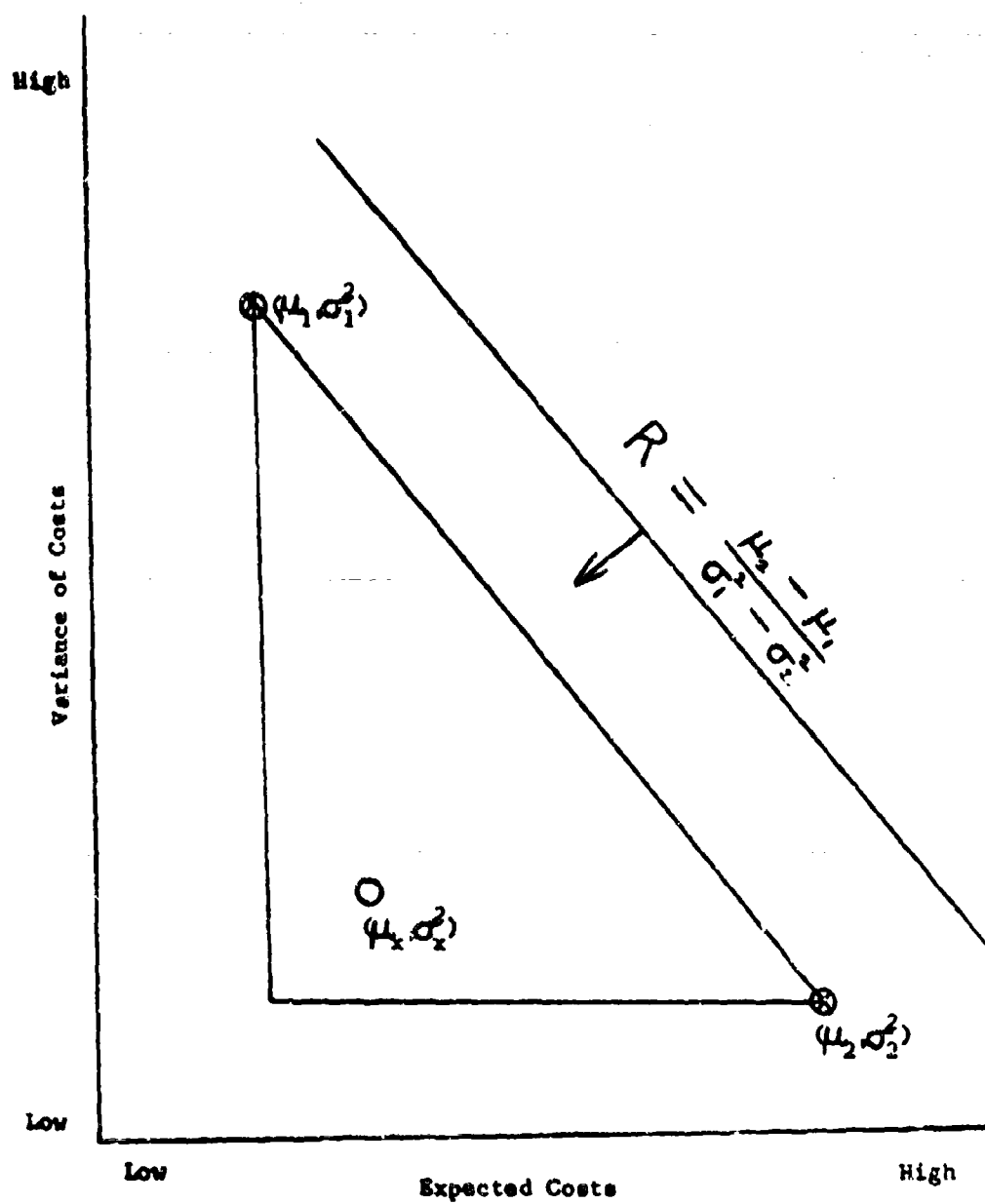


Figure 5.6

Testing for a Skipped FEMD System

to be the FEMD system for that R, but not both. A perturbation of R would indicate the alternative FEMD system.

Applying the testing procedure to the two-subsystem example in order to check for a FEMD system between the R values of 0.0 and 1.0, we use as the trial value

$$\begin{aligned} R &= \frac{\mu_1 - \mu_{16}}{\sigma_{16}^2 - \sigma_1^2} \\ &= \frac{2,280 - 2,090}{230,000 - 80,000} \\ &= 0.0013. \end{aligned}$$

Of all the feasible systems, system 9 ($\mu = 2160$, $\sigma^2 = 120,000$) has minimum expected disutility for $R = 0.0013$ and an unknown FEMD system is revealed.

We now have two ranges of R which require a second range test. Testing these two ranges, for $0.0 < R < 0.0013$ try

$$R = \frac{2,160 - 2,090}{230,000 - 120,000} = 0.0006, \text{ and for } 0.0013 < R < 1.0 \text{ try}$$

$$R = \frac{2,280 - 2,160}{120,000 - 80,000} = 0.003. \text{ These values of R produce two new FEMD systems; namely, system 11 } (\mu = 2120, \sigma^2 = 160,000) \text{ and system 5 } (\mu = 2215, \sigma^2 = 100,000).$$

There are now four ranges of R which require a third range test. These are: $0.0 < R < 0.0006$, $0.0006 < R < 0.0013$, $0.0013 < R < 0.003$, and $0.003 < R < 1.0$. For these four ranges of R, we try R values of 0.0004, 0.001, 0.0028 and 0.0033 respectively. These values of R produce a new FEMD system, system 15 ($\mu = 2110$,

$\sigma_2^2 = 180,000$), for $R = 0.0004$ but produce repeats of already discovered FEMD systems for the other three R values. (We prove in the next subsection that no new FEMD systems can exist between R 's which have the same FEMD system.)

Thus, we have only two ranges of R which require a fourth range test; $0.0 < R < 0.0004$ and $0.0004 < R < 0.0006$. Computing our trial values of R for these ranges we have $R = 0.0004$ (a value already tested) and $R = 0.0005$. Thus, we need test only $R = 0.0005$ and since this R values produces a FEMD system previously discovered, the generation of the entire set of FEMD systems (the schedule of efficient systems) is complete. Table 5.1 summarizes the FEMD generation results, with redundant systems indicated with their corresponding R values.

Testing between R 's which have the same FEMD system. When a change from one value of R to another value of R results in generation of the same FEMD system, one might suspect that trying intermediate values of R between these two values would yield no new FEMD system. In our two-subsystem example, the ranges $0.0006 < R < 0.0013$, $0.0013 < R < 0.003$, $0.003 < R < 1.0$, and $0.0004 < R < 0.0006$ exemplify this case. In general, if the FEMD system S_1 corresponding to $R = R_1$ is the same as the FEMD system S_2 corresponding to $R = R_2$, much computational effort might be avoided if all values of R_x could be spared investigation where $R_1 < R_x < R_2$. Indeed a theorem will be stated and proved which permits an even more

Table 5.1

FEMD Systems for Two-
Subsystem Example

Test No.	Coeff. of Risk Aver. ($\times 10^6$)	μ ($\times 10^{-6}$)	σ^2 ($\times 10^{-15}$)	System
1	0.0	2090	230	16
5	400	2110	180	15*
4	400	2110	180	15
5	500	2110	180	15*
3	600	2120	160	11
4	1000	2120	160	11*
2	1300	2160	120	9
4	2800	2160	120	9*
3	3000	2215	100	5
4	3300	2215	100	5*
1	1×10^6	2280	80	1

* Redundant systems (that is, FEMD systems previously discovered).

general conclusion. Namely, the subsystem alternative constituents of S_1 and S_2 need not even be the same; only expected cost equality and variance equality of the two systems is necessary in order to ensure the non-existence of a system S_x having less disutility than either S_1 or S_2 .

Theorem: For any R such that $R_1 < R_x < R_2$, if system S_1 has the same expected cost and the same variance as does system S_2 , (that is, $\mu_1 = \mu_2 = \mu$ and $\sigma_1^2 = \sigma_2^2 = \sigma^2$) then the expected cost μ_x and variance σ_x^2 of system S_x are equal respectively to μ and σ^2 .

Proof: For any three FEMD systems S_1 , S_x , and S_2 where each has minimum expected disutility at its corresponding coefficient of risk,

$$(1) \quad \mu_x + R_1 \sigma_x^2 \geq \mu_1 + R_1 \sigma_1^2$$

$$(2) \quad \mu_1 + R_x \sigma_1^2 \geq \mu_x + R_x \sigma_x^2$$

$$(3) \quad \mu_x + R_2 \sigma_x^2 \geq \mu_2 + R_2 \sigma_2^2$$

For $\mu_1 = \mu_2 = \mu$ and $\sigma_1^2 = \sigma_2^2 = \sigma^2$, these relationships become

$$(4) \quad \mu_x + R_1 \sigma_x^2 \geq \mu + R_1 \sigma^2$$

$$(5) \quad \mu + R_x \sigma^2 \geq \mu_x + R_x \sigma_x^2$$

$$(6) \quad \mu_x + R_2 \sigma_x^2 \geq \mu + R_2 \sigma^2$$

Consider (4) + (5):

$$\mu + \mu_x + R_1 \sigma_x^2 + R_x \sigma^2 \geq \mu + \mu_x + R_1 \sigma^2 + R_x \sigma_x^2,$$

or

$$R_1(\sigma_x^2 - \sigma^2) \geq R_x(\sigma_x^2 - \sigma^2)$$

which is true only when

$$\sigma_x^2 \leq \sigma^2 \text{ since } R_1 < R_x.$$

Consider (5) + (6)

$$\mu + \mu_x + R_2 \sigma_x^2 + R_x \sigma^2 \geq \mu + \mu_x + R_2 \sigma^2 + R_x \sigma_x^2$$

or

$$R_2(\sigma_x^2 - \sigma^2) \geq R_x(\sigma_x^2 - \sigma^2)$$

which is true only when $\sigma_x^2 \geq \sigma^2$ since $R_x < R_2$. Therefore the strict inequality is contradicted, and the equality holds.

Consider (4) when $\sigma_x^2 = \sigma^2$:

$$\mu_x + R_1 \sigma^2 \geq \mu + R_1 \sigma^2,$$

or

$$\mu_x \geq \mu$$

Consider (5) when $\sigma_x^2 = \sigma^2$:

$$\mu + R_x \sigma^2 \geq \mu_x + R_x \sigma^2,$$

or

$$\mu \geq \mu_x$$

and again the equality holds. Thus, the expected return and variance of S_x are equal respectively to μ and σ^2 , and the proof is complete.

Summary of the Search Procedure

The search procedure described in the preceeding paragraphs can be summarized as follows: initially investigate the end point values of R , that is, $R = 0.0$ and $R \rightarrow \infty$ (for the examples presented in Chapter VI, $R = 1.0$ is sufficiently large enough to encompass the upper extreme risk aversion point) and then systematically obtain additional FEMD systems, if they exist, by the successive testing method previously outlined. As illustrated in Chapter VI, the number of successive tests required to generate the entire set of FEMD systems is quite small (nine successive tests being the maximum required for the least constrained problem studied). In actuality the FEMD systems generated in the successive tests are obtained by a computer programmed mathematical programming technique. The model user need only compute the trial values of the R 's for each successive test as previously outlined.

RELIABILITY/MAINTAINABILITY CONCEPTUAL
DESIGN MODEL COMPUTER PROGRAM

The computer program is written in FORTRAN IV and allows for consideration of up to 180 variables and 40 constraints. The program as currently dimensioned uses 62,000 words of core on the Honeywell 600 computer. In terms of our reliability/maintainability model, the program is dimensioned to handle a system with up to twenty subsystems with nine alternatives per subsystem.

The program is composed of a main program and two subroutines. The main program converts the input data information into the required zero-one linear programming formulation. It also converts the final solution into the final summary solution report provided by the program (see Appendix B). The first subroutine program, Subroutine RAM, contains the multiple-choice implicit enumeration solution algorithm. It accepts the zero-one linear programming

problem from the main program and solves it. The second subroutine, Subroutine SIMPLE, is a linear programming subroutine which is used for generating our Step 2 surrogate constraints.

In this section we will discuss the computer program, computer input requirements, and computer outputs. A description of Subroutine RAM will also be presented in the form of a block diagram. A complete listing of the program is contained in Appendix A.

Main Program

The main program performs two basic functions: it prepares the required zero-one linear programming formulation of the problem and prepares the final summary report printout. The inputs to the main program are: a set of parameter values which stipulate control values for the particular problem to be run, standard normal deviate values for each external chance-constraint, desired levels for each external constraint, risk aversion coefficient trial values, and the input mean and variance estimates for each subsystem alternative. The subsystem alternative mean and variance estimates required are those listed in the input data matrix illustrated in Figure 3.2 page 51.

No input data values are required for the dependent and internal constraints. These constraints are formulated directly in the main program and are controlled by input parameter values.

Since the external constraints are chance-constraints, the program converts the input mean and variance estimates into the

linearized deterministic equivalent forms derived in Chapter IV and transforms these into the required constraint form.⁴³ The external constraints for weight, R&D cost, and failure rate (2), (3) and (4) pp. 60-61) do not involve functions of random variables. Thus, the conversions for these three external constraints are straight forward. The availability and maintainability external constraints ((1) and (5) pp. 60-61) do involve functions of random variables. Thus, their conversions require that we first find means and variances for their random variable functions. Both constraints contain functions which involve the product of independent random variables (that is, $\lambda_{ij} \cdot \text{MTTR}_{ij}$). To find the mean and variance of this product we use the following property of the product of independent variables: if X_1 and X_2 are independent random variables and

$$u = X_1 X_2$$

then

$$E[u] = \mu_1 \mu_2$$

$$\text{Var}[u] = \sigma_1^2 \sigma_2^2 + \mu_1^2 \sigma_2^2 + \mu_2^2 \sigma_1^2$$

In addition to the product of random variables, the availability constraint includes a term of the form

$$\ln[a_1 - a_2 u]$$

where a_1 and a_2 are constants and $u = \lambda_{ij} \cdot \text{MTTR}_{ij}$. The mean

⁴³The following general zero-one linear programming form is required by the solution subroutine, Subroutine RAM: Minimize $\sum C X$ subject to $b + A X \geq 0$ where C and X are n -vectors (n = number of subsystems \cdot number of alternatives per subsystem), b is an m -vector (m = total number of constraints), and A is m by n .

and variance of the function inside the brackets is $a_1 - a_2 E[u]$ and $a_2^2 \text{Var}[u]$ respectively. Letting $w = a_1 - a_2 u$, our problem is to find the mean and variance of $\ln(w)$ when the form of the random variable w is unknown and its mean and variance are as specified in the previous sentence. Since the form of w is unknown, we must use an approximation technique to determine its expected value and variance. A Taylor's series expansion of $\ln(w)$ about its expected value μ yields

$$\ln(w) = \ln(\mu) + \frac{1}{\mu} (w - \mu) - \frac{1}{2\mu^2} \frac{(w - \mu)^2}{2!} + \dots$$

Using the first three terms of the infinite series as an approximation for $\ln(w)$ and recalling that $E[w] - \mu = 0$ and $E[(w - \mu)^2] = \sigma^2$, then

$$E[\ln(w)] = \ln(\mu) - \frac{\sigma^2}{2\mu^2}$$

and

$$\begin{aligned} \text{Var}[\ln(w)] &= \text{Var}\left[\frac{w - \mu}{\mu}\right] + \text{Var}\left[\frac{(w - \mu)^2}{2\mu^2}\right] \approx \frac{\text{Var}[w]}{\mu^2} \\ &= \frac{\sigma^2}{\mu^2}, \text{ if we neglect} \end{aligned}$$

the second term in our variance approximation.⁴⁴ These approximations are programmed into the main program and are used for converting the chance-constrained availability constraint to its deterministic linearized equivalent form.

⁴⁴Shoeman(48p. 414) shows that it is not unreasonable to approximate the variance with only the first term of the variance of the Taylor series.

Upon completion of the formulation of the external constraints, any dependence constraints that may exist and the internal constraints are formulated. Then the expected disutility life cycle cost function is computed for a particular trial value of the coefficient of risk aversion, R . The complete zero-one linear programming problem is then transferred to Subroutine RAM for solution. When a final solution is found, it is transferred back to the main program where the solution is used to generate the summary report printout. The main program then checks to see if another trial value of R is to be tested. If another R is to be tested, a new expected disutility life cycle cost function is computed for the new R value and this new function along with the previously formulated constraints are sent to Subroutine RAM for solution. The process repeats itself until all input R trial values have been run.

Subroutine RAM

The multiple-choice zero-one linear programming solution algorithm is programmed as Subroutine RAM in the computer program. Details of this subroutine are presented in Figure 5.7 in block diagram form.

Subroutine SIMPLE

The linear programming subroutine is basically a Revised Simplex method with explicit inverse, the starting point having been a routine due to Clasen(117). Restarting techniques are incorporated that use a labeling procedure rather than more conventional matrix

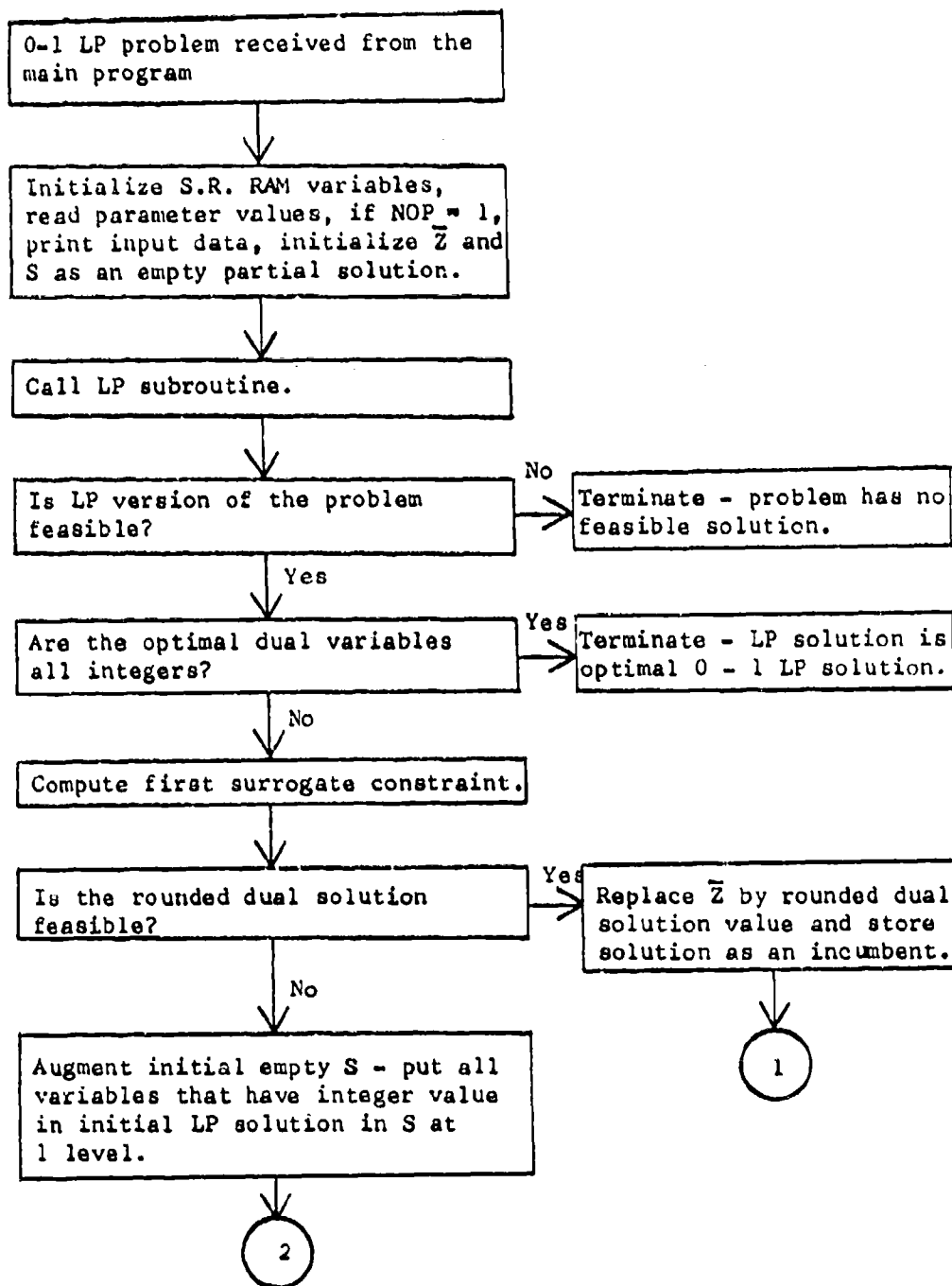


Figure 5.7

Block Diagram of Subroutine RAM

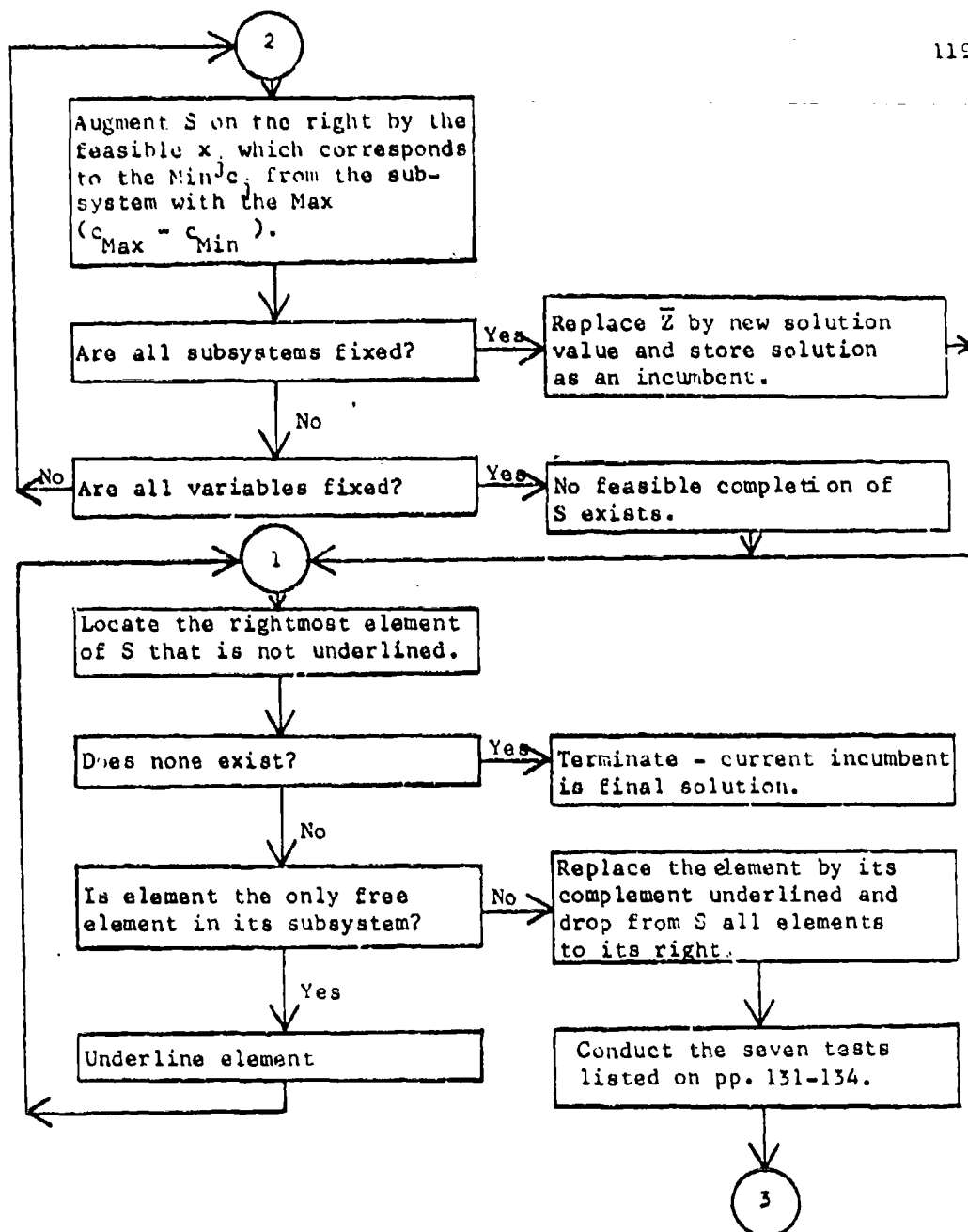


Figure 5.7 (continued)

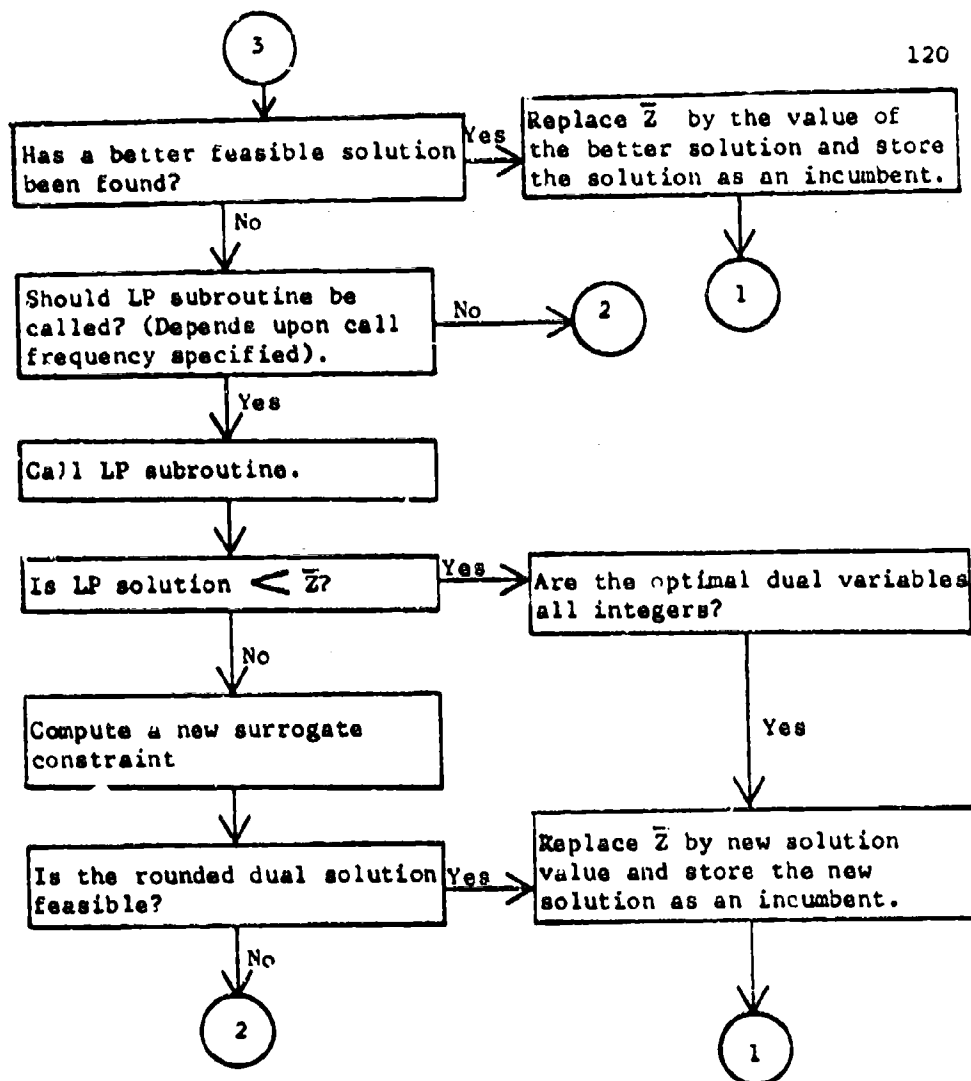


Figure 5.7 (continued)

manipulations. The labeling procedure is based on the observation that fixing a variable at the value 0 or 1 can be viewed as demanding equality in the appropriate inequality constraint among $0 \leq x_j \leq 1$, $j \in s$, in the continuous version of (P_g) . This means that the corresponding dual variables (the w_j and slacks in (LP_g)) become unconstrained in sign; the appropriate variables are therefore labeled and treated as "unsigned." This procedure is easier to program than a more conventional one using matrix manipulations, and has the advantage of being economical in terms of core and setup time for the successive linear programs. It has the drawback, however, that (LP_g) (and therefore the explicit inverse) always has n rows, instead of only as many rows as free variables. Hence, each pivot requires more work.

The actual subroutine program used is essentially the same as that used by Geoffrion and Nelson (129). Changes were made as required to correct errors in the subroutine as originally written.

Computer Program Input Requirements

The computer program requires the following parameter and data cards for each run (a run may consist of multiple problems for various R trial values):

Main program parameter card. The main program input parameters are:

ISS - The number of subsystems.

N - The number of variables. N is equal to the

number of subsystems times the number of alternatives per subsystem.

M - The number of constraints. M is equal to the number of external constraints plus the number of dependence constraints plus the number of internal constraints ($ISS + 1$).

ICON - The number of alternative input data sets to be input into the program. We input five sets - our life cycle cost, R&D cost, weight, failure rate, and maintainability mean and variance estimates - for each alternative for each subsystem.

ICON: 1 - The number of external constraints having all negative a_{ij} 's. This would include all external constraints but the maintainability constraints.

IRISK - The number of trial values of R to be run.

IVER - A code which specifies which external constraints are to be used in the problem to be run.

The code values and their meaning are:

1 - Run problem with availability and weight constraints only.

2 - Run problem with R&D cost, availability, and weight constraints.

3 - Run problem with failure rate, maintainability, and weight constraints.

4 - Run problem with R&D cost, failure rate, maintainability, and weight constraints.

5 - Run problem with R&D cost, availability, failure rate, maintainability, and weight constraints.

6 - Run problem to minimize R&D cost subject to failure rate, maintainability, and weight constraints.

IDEP - A control variable which indicates whether or not dependence constraints exist. If IDEP = 0, there are no dependence constraints. If IDEP = 1, there are dependence constraints in the problem.

The format for all parameter values is I3.

Standard normal deviate card. This card contains the standard normal deviates corresponding to the probability restrictions of the external chance-constraints. A separate deviate value is entered for each external chance-constraint. The format for this card is 6F12.4.

External constraints limit card. This card contains the limits stipulated for each external chance-constraint. The format for this card is 6F12.4.

Coefficient of risk aversion card. The trial values of the coefficient of risk aversion are listed on this card. The format for this card is 6F12.4.

Input data estimates cards. The input mean and variance estimates are read in via these sets of cards. Each card contains mean and variance estimate values for three alternatives of a subsystem. The values are punched in F12.4 fields in pairs (that is, a mean and variance pair). The first set of these cards contain the life cycle cost estimates for each alternative for each subsystem. The succeeding sets arranged in order are: the R&D cost estimates, the failure rate estimates, the weight estimates, and the maintainability estimates.

Subroutine RAM parameter card. The input parameters for the solution algorithm are input on the last data card. These parameter are:

M - The number of constraints.

N - The number of variables.

ISS - The number of subsystems.

L - If $L = 0$, the initial partial solution is empty and the first LP solution is not used to create the next partial set. If $L < 0$, an "LP start" is used and the integer variables from the first LP solution are used as the next partial set.

SC - If $SC = 0$, no imbedded linear program is desired (the algorithm then eliminates Step 2 and does not use surrogate constraints). If $SC = 1$, the imbedded linear program is to be used.

ZBAR - If an upper bound \bar{Z} on the optimal value of the objective function of (P) is known, put $ZBAR = \bar{Z} - gcd + 0.0001$, where gcd is the greatest common divisor of the cost coefficients c_j . Hence, if all c_j are integer, put $ZBAR = \bar{Z} - 0.9999$. The effect will be that the program looks only for feasible solutions with value $< ZBAR$. If no upper bound is known, put $ZBAR = 0$.

ISQMAX - The maximum number of surrogate constraints that will be carried. $ISQMAX = 4$ is reasonable.

ISCFR - The frequency with which the imbedded linear program is used. $ISCFR = 0$ means that it will never be used; $ISCFR = j$, j a positive integer, means that it be used every j th time. $ISCFR = 8$ has proven effective.

NOP - If NOP = 1, the objective function

c_j 's and the b_i and a_{ij} values for the external constraints will be printed out. If NOP = 2, the partial solution j values will be printed each time the LP subroutine is called. If NOP = 3, the surrogate constraints will be printed out as they are generated. If NOP $>$ 3, only the normal output will be printed out (see Appendix B).

ZKBAR - ZKBAR = gcd - 1 (see ZBAR). Thus if all c_j are integer, put ZKBAR = 0. The effect is that the program looks only for feasible solutions with value at least (ZKBAR + .99999) less than the best feasible solution currently known; this doesn't exclude any optimal solutions. (A solution within Δ of the optimum can be found if desired by increasing the above value of ZKBAR by Δ .)

H1,H2 - Arbitrary problem identifiers.

The format for this parameter card is: 5I3, F12.4, 3I3, F12.4, 2A6.

Computer Output Interpretation

Execution of the computer program produces three types of final output plus the intermediate output controlled by the NOP

input parameter previously discussed. Since the intermediate output (that is, the output controlled by NOP) is likely to be of little incremental value to the user over the final output information, no detailed explanation other than that given in the previous subsection is given here.

The first type of final output is produced as Subroutine RAM is being executed, and the other two types of final output are produced after a final solution has been found. Each solution for each trial coefficient of risk aversion produces a complete set of all three types of final output. Appendix B contains an example listing of the final output from one of the Chapter VI example runs.

The first type of final output gives the following: the value of the linear programming solution of the problem and a listing of each feasible solution found (objective function value and a list of which variables equal 1) during the execution of the solution algorithm. This output is provided by Subroutine RAM.

The second type of final output is also provided by Subroutine RAM. The output given is the following: the value of the trial coefficient of risk aversion, R ; the number of feasible solutions found; the number of times the imbedded linear program was called; the number of iterations; the value of the optimal solution; and a listing of the solution variables. The listing is presented in matrix form, where the rows represent the subsystems and the columns the alternatives. A zero in a matrix cell means that that subsystem alternative was not selected. A

positive integer j in a cell means that that subsystem alternative was selected.

The third type of final output, the summary printout, is prepared in the main program. The main program receives the final solution from Subroutine RAM and uses the solution to compile overall system results. The specific information given is: the coefficient of risk aversion; the desired external constraint levels for those external constraints used in the particular problem; the expected value, variance, and standard deviation for the system life cycle costs, R and D costs, failure rate, weight, maintainability, and availability; and the subsystem alternative levels selected for the optimal system for the particular trial value used for the coefficient of risk aversion. Examples of type three final output data is given in Tables 5.2 and 5.3.

Table 5.2

Type-three Final Output

RESULTS FOR RISK LEVEL 1

COEFFICIENT OF RISK = 0.0004700
MAXIMUM FAILURE RATE = 8500.0
MAXIMUM WEIGHT = 6000.0
MAXIMUM MEAN TIME TO REPAIR = 109.0

THE OPTIMAL SYSTEM RESULTS ARE

EXPECTED LIFE CYCLE COSTS = 13467.5
VARIANCE OF LIFE CYCLE COSTS = 2031250.0
STANDARD DEVIATION OF LIFE CYCLE COSTS = 1425.2

EXPECTED R AND D COSTS = 3462.5
VARIANCE OF R AND D COSTS = 48920.0
STANDARD DEVIATION OF R AND D COSTS = 221.2

EXPECTED FAILURE RATE = 7675.0
VARIANCE OF FAILURE RATE = 111950.0
STANDARD DEVIATION OF FAILURE RATE = 334.6

EXPECTED WEIGHT = 4973.0
VARIANCE OF WEIGHT = 69899.0
STANDARD DEVIATION OF WEIGHT = 264.4

EXPECTED MEAN TIME TO REPAIR = 156.1
VARIANCE OF MEAN TIME TO REPAIR = 152.7
STANDARD DEVIATION OF MTR = 12.4

EXPECTED AVAILABILITY = 0.9611
VARIANCE OF AVAILABILITY = 0.00000937
STANDARD DEVIATION OF AVAILABILITY = 0.0031

Table 5.3

Type-Three Final Output

THE OPTIMAL SUBSYSTEM ALLOCATIONS ARE

SUBSYSTEM	1	LEVEL IS = 6
SUBSYSTEM	2	LEVEL IS = 5
SUBSYSTEM	3	LEVEL IS = 5
SUBSYSTEM	4	LEVEL IS = 6
SUBSYSTEM	5	LEVEL IS = 8
SUBSYSTEM	6	LEVEL IS = 5
SUBSYSTEM	7	LEVEL IS = 6
SUBSYSTEM	8	LEVEL IS = 6
SUBSYSTEM	9	LEVEL IS = 5
SUBSYSTEM	10	LEVEL IS = 4
SUBSYSTEM	11	LEVEL IS = 3
SUBSYSTEM	12	LEVEL IS = 6
SUBSYSTEM	13	LEVEL IS = 5
SUBSYSTEM	14	LEVEL IS = 6
SUBSYSTEM	15	LEVEL IS = 7
SUBSYSTEM	16	LEVEL IS = 5
SUBSYSTEM	17	LEVEL IS = 9
SUBSYSTEM	18	LEVEL IS = 5
SUBSYSTEM	19	LEVEL IS = 4
SUBSYSTEM	20	LEVEL IS = 3

CHAPTER VI

SOLUTION AND DISCUSSION OF
SYSTEM SELECTION EXAMPLES

In this chapter, solutions of the model for a system comprised of twenty subsystems are obtained and discussed. Schedules of efficient systems are generated under the assumptions of (1) risk indifference and deterministic constraints; (2) risk consideration and availability and weight chance-constraints; (3) risk consideration and availability, weight, and R&D budget chance-constraints; and (4) risk consideration and reliability, maintainability, and weight chance-constraints. All examples include consideration of dependence among specific subsystem alternatives. Final system selection from a schedule of efficient systems is also discussed. The chapter is concluded with a summary of computational time statistics for the computer runs of the example problems.

It should be recognized that although the actual results given here are dependent on the input data, the methods should be readily adaptable to a variety of such problems, and should therefore be of general interest.

UTILITY OF THE MODEL

To demonstrate the utility of the model and solution technique complete schedules of efficient systems are generated for three

external constraint combinations. The first constraint combination imposed includes only the basic constraints found in the conceptual acquisition model; that is, system availability and weight chance-constraints. Next we add an R&D cost chance-constraints to illustrate the effect that the imposition of such a constraint has on the schedule previously generated. The final combination, for which we generate a complete schedule of efficient systems, contains system reliability, maintainability, and weight chance-constraints. This combination was chosen so that we could illustrate the effect of considering reliability and maintainability separately instead of combined as a measure of system availability. We also add an R&D cost constraint to our final combination and obtain results for selected risk aversion coefficient trial values.

In all examples, mutually-contingent dependence is assumed to exist among the ultra high reliability alternatives (that is, alternatives 7, 8, and 9) for subsystems 18, 19, and 20. This dependence is included in the example problems by imposing the constraint $2x_{160} + 2x_{161} + 2x_{162} - x_{169} - x_{170} - x_{171} - x_{178} - x_{179} - x_{180} = 0$. Also, mutually-exclusive dependence is assumed to exist, in all examples, among the ultra high maintainability alternatives (that is, alternatives 3, 6, and 9) for subsystems 13, 15, and 16. The constraint imposed to reflect this dependence is $1 - x_{111} - x_{114} - x_{117} - x_{129} - x_{132} - x_{135} - x_{138} - x_{141} - x_{144} \geq 0$.

Data Input Used for the Examples.

An example of the subsystem input data used for the example problems is given in Table 6.1. Appendix C contains a listing of the input data for all twenty subsystems. Although the data used is hypothetical, it was not randomly selected, but was carefully chosen to reflect reasonable relationships among the various subsystem data elements. The specific values used were chosen to reflect the three observations pertaining to reliability/maintainability attainment and cost discussed in Chapter II, pages 21-23. Also, the dependence which exists between reliability and maintainability within a subsystem is reflected in the data. Specifically, it is assumed that for a given reliability level, maintainability improvements tend to degrade subsystem reliability and for a given maintainability level, reliability improvements tend to degrade subsystem maintainability.

The dimensions used for the various data elements are given in Table 6.1. The life cycle cost data elements are based upon 700 equipment units (aircraft, in our case) operating thirty hours per month each over a ten year life cycle. The failure rate is given in total force failures per month and the weight data elements are the additional pounds added to the subsystem if the specific alternative is selected.

Since the computer program requires that all subsystems have an equal number of alternatives (a situation that probably would not exist in practice), redundant alternatives are added for those

TABLE 6.1
DATA INPUT FOR SUBSYSTEM 1

DATA INPUT FOR SUBSYSTEM 1						
LUC	EXPECTED VALUE ($\times 10^{-6}$)	VARIANCE ($\times 2 \times 10^{-12}$)	R	AND U	EXPECTED VALUE ($\times 10^{-6}$)	VARIANCE ($\times 2 \times 10^{-12}$)
	590.0	10000.0			30.0	160.0
	557.0	15000.0			53.0	250.0
	530.0	20000.0			78.0	420.0
	500.0	25000.0			75.0	500.0
	525.0	38750.0			90.0	750.0
	505.0	62200.0			105.0	950.0
	540.0	95000.0			130.0	750.0
	525.0	150000.0			140.0	900.0
	500.0	202500.0			160.0	1300.0
W	EXPECTED VALUE (LBS)	VARIANCE (LBS ²)	FK	EXPECTED VALUE (FAILURES/MONTH)	VARIANCE (FAIL ² /MN ²)	
	50.0	300.0		100.0	200.0	
	80.0	500.0		110.0	250.0	
	100.0	670.0		130.0	350.0	
	100.0	450.0		75.0	250.0	
	125.0	700.0		85.0	300.0	
	140.0	1200.0		105.0	400.0	
	210.0	3600.0		50.0	350.0	
	230.0	4000.0		60.0	400.0	
	250.0	5000.0		80.0	500.0	
MTR	EXPECTED VALUE (MINUTES)	VARIANCE (MIN ²)				
	75.0	30.0				
	45.0	50.0				
	30.0	100.0				
	150.0	150.0				
	105.0	250.0				
	75.0	500.0				
	300.0	1000.0				
	240.0	1800.0				
	210.0	3600.0				

subsystems having less alternatives than the maximum of the other subsystems. In our data, all subsystems except subsystems 5, 14, 17, and 19 have nine alternatives. Thus, we repeat some of the alternatives that these subsystems do have so that they appear to have the same number of alternatives as the other subsystems. This has no effect on the solution but it does facilitate programming the solution algorithm.

Risk Indifference and Deterministic Constraints

Setting

the coefficient of risk aversion, R , equal to zero implies risk indifference since this reduces the model's objective function to minimization of system expected life cycle cost. Deterministic constraints are produced by setting the standard normal deviate for each external constraint imposed equal to zero. This causes the chance-constraints to reduce to simple expected value constraints of the form

$$\sum_i \sum_j E[a_{ij}] x_{ij} \geq \text{or} \leq b_h.$$

Under these conditions, the schedule of efficient systems generated by solving the model consists of the single system which minimizes system expected life cycle cost. Solutions of the model under risk indifference and various combinations of deterministic external constraints are given in Table 6.2. The b_h values stipulated for the various constraints were:

Availability	\geq	0.95
Weight	\leq	6000 pounds

Table 6.2
Risk Indifference and Deterministic Constraints

Constraints Imposed	E (GOC) (\$ 10^{-6})	Var (LCC) (\$ 10^{-15})	STD. DEV. (LCC) (\$ 10^{-6})	Subsystem Alternatives Selected
Availability and Weight	13047.5	4514.0	2124.6	6,4,3,3,3,5,9,8,6,9,8,6,9,9,8,7,8,8,9
Reliability, Maintainability and Weight	13055.0	4432.5	2106.0	6,4,3,6,3,5,9,8,6,9,8,6,9,8,8,7,9,8,9
Availability, Weight and R&D Cost	13227.5	3535.25	1880.25	6,8,3,6,3,5,5,6,8,5,5,9,8,4,8,7,8,7,8
Reliability, Maintainability, Weight and R&D Cost	13230.0	3572.75	1890.15	6,8,3,6,3,5,6,8,8,9,5,5,9,8,2,8,9,8,7,9
Reliability, Maintainability, and Weight ³⁵	14965.0	792.5	890.2	1,7,4,1,7,4,1,4,4,1,1,1,7,1,1,2,1,4,4,1

³⁵ The objective function used for this case was minimization of expected R&D cost.

R&D Cost \leq \$3,500 million
 Reliability \leq 8500 failures/month
 Maintainability \leq 3.15 hours.

The solutions listed for the first four constraint combinations provide those combinations of subsystem alternatives which minimize the system's expected life cycle cost. These systems would appear attractive to a decision-maker who is unconcerned about the variability of system life cycle cost and unconcerned that the probability of not satisfying any one of the imposed constraints could be as high as 0.50 (since for the normal distribution, $a_h = .50$ when $z_h = 0$). The fifth solution listed provides that combination of subsystem alternatives which minimizes the expected R&D cost for the system. The relationships of these risk indifferent and deterministic constraint efficient systems to those in schedules generated under risk conditions is discussed in the succeeding parts of this section.

Risk Considered and Chance-Constraints

We now provide solutions obtained from our model (schedules of efficient systems) for the decision-maker who is not indifferent to risk. He is concerned about both the risk and cost of the systems being considered and is not satisfied with a 0.50 probability that his constraints will be violated. First, we impose only the basic constraints included in the acquisition problem conceptual model developed in Chapter II; that is, constraints on system availability and

system weight. Then we impose other restrictions (R&D cost, reliability, and maintainability) which are frequently imposed in practice and examine the effect of these other restrictions.

Availability and weight chance-constraints. The schedule of efficient systems, generated with risk considered and external constraints imposed which require system availability ≥ 0.95 and the increase in system weight ≤ 6000 pounds with probability ≥ 0.95 in each case, is listed in Table 6.3. This schedule was generated by minimizing our expectation-variance disutility function for trial value of R chosen in accordance with the search procedure described in Chapter V. Only thirty-one of the possible $(9)^{20}$ system combinations are included in the schedule of efficient systems generated; all other combinations being eliminated on the basis of feasibility or efficiency. All of the thirty-one systems included in the schedule satisfy the imposed availability, weight and dependence constraints and any one of them could be suitable to the decision-maker depending upon his preferences regarding cost and risks.

If we draw a curve through the expectation-variance points representing these thirty-one systems as shown in Figure 6.1, we form the lower left-hand boundary of the region of feasible systems. No feasible systems lie to the left and below this boundary. If some did, they would be more efficient (in an expectation-variance sense) than the systems in our schedule and would replace those in our schedule and form a new boundary to the left and lower

Table 6.3

Risk Considered and Availability
and Weight Chance-Constraints

Coeff. of Risk Aver. ($\text{RX}10^6$)	E (LCC) ($\text{\$X}10^{-6}$)	VAR (LCC) ($\text{\$X}10^{-15}$)	STD. DEV. (LCC) ($\text{\$X}10^{-6}$)	Subsystem Altern. Selected (SS-1 to 10) (SS-11 to 20)
0	13162.5	3954.0	1988.45	6,4,3,9,3,5,9,8,6,9 3,3,5,8,8,9,7,8,8,9
22	13172.5	3425.25	1850.75	6,4,3,6,6,5,6,8,6,9 3,6,5,7,8,9,7,8,8,9
42	13177.5	3247.75	1802.15	6,4,3,6,9,5,6,8,6,9 3,6,5,8,8,9,7,8,7,9
70	13185.0	3132.75	1769.95	6,4,3,6,6,5,6,8,6,9 5,6,5,7,9,8,9,8,7,9
98	13190.0	3070.0	1752.15	6,5,3,6,9,5,6,8,6,9 3,6,5,7,9,8,9,8,7,9
186	13205.0	2965.0	1721.90	6,5,3,6,9,5,6,8,6,9 3,6,5,7,9,7,9,8,7,9
222	13220.0	2890.0	1700.0	6,5,3,6,9,5,6,8,6,9 3,6,5,7,9,7,7,7,7,9
230	13270.0	2672.50	1634.8	6,5,3,6,9,5,6,8,6,9 3,6,5,7,7,7,7,7,7,9
274	13300.0	2552.50	1597.65	6,5,3,6,9,5,6,8,6,9 3,6,5,6,7,7,9,7,7,9
278	13392.5	2215.0	1488.30	6,5,3,6,9,5,6,8,6,9 3,6,5,5,7,7,7,3,4,3
300	13427.5	2095.0	1447.40	6,5,3,6,6,5,6,8,6,9 3,6,5,6,5,7,7,3,4,3
316	13447.5	2030.0	1424.80	6,5,3,6,8,5,6,8,6,9 3,6,5,5,5,7,9,3,4,3

Table 6.3 (continued)

Coeff. of Risk Aver. ($\times 10^6$)	E (LCC) (\$ $\times 10^{-6}$)	VAR(LCC) (\$ $\times 10^{-15}$)	STD. DEV.(LCC) (\$ $\times 10^{-6}$)	Subsystem Altern. Selected (SS-1 to 10) (SS-11 to 20)
350	13460.0	1992.50	1411.55	6,5,3,6,8,5,6,8,6,9 3,6,3,5,5,7,9,3,4,3
400	13500.0	1887.50	1373.85	6,5,5,6,8,5,6,3,6,9 3,6,3,5,5,7,9,3,4,3
408	13587.5	1672.50	1293.25	6,5,3,6,8,5,6,3,6,9 3,3,3,5,5,5,7,3,4,3
478	13600.0	1643.75	1282.10	6,5,3,6,8,5,6,3,5,9 3,3,3,5,5,5,7,3,4,3
496	13685.0	1468.75	1211.90	6,5,3,6,8,5,6,3,5,7 3,3,3,5,5,5,7,3,4,3
510	13695.0	1448.75	1203.65	6,5,3,6,2,5,5,3,5,7 3,3,3,5,5,5,7,3,4,3
528	13717.5	1405.0	1185.35	6,5,3,3,8,5,5,3,5,7 3,3,3,5,5,5,7,3,4,3
602	13742.5	1359.75	1166.10	3,5,3,3,2,5,5,3,5,7 3,3,3,6,5,5,7,3,4,3
666	13880.0	1141.0	1068.20	3,5,3,3,2,5,5,3,6,7 3,3,3,5,5,5,4,3,4,3
883	14032.5	924.75	961.65	3,5,3,3,2,5,5,3,5,3 3,3,3,5,5,5,4,3,4,3
912	14147.5	797.25	892.90	3,5,3,3,2,5,5,3,5,3 2,3,2,2,3,5,4,3,4,3
1010	14162.5	782.25	884.45	3,5,3,3,2,5,5,3,3,3 3,3,2,2,3,5,4,3,4,3
1146	14220.0	729.75	854.25	3,5,3,3,8,5,3,3,3,3 2,3,2,2,3,5,4,3,4,3

Table 6.3 (continued)

Coeff. of Risk Aver. (RX10 ⁶)	E (LCC) (\$X10 ⁻⁶)	VAR (LCC) (\$X10 ⁻¹⁵)	STD. DEV. (LCC) (\$X10 ⁻⁶)	Subsystem Altern. Selected (SS-1 to 10) (SS-11 to 20)
1200	14275.0	683.50	826.75	3,5,3,3,1,5,3,3,3,3 2,3,2,2,3,5,4,3,4,3
1602	14280.0	679.50	824.45	3,5,3,3,1,5,2,3,3,3 2,3,2,2,3,5,4,3,4,3
3936	14355.0	646.25	803.90	3,3,5,3,7,3,2,3,3,3 2,3,2,3,3,4,4,3,4,3
5158	14427.5	631.25	794.50	3,3,4,2,7,3,2,3,3,2 1,3,2,2,3,5,4,3,4,3
5914	14477.5	622.50	769.0	2,3,5,2,1,1,2,3,3,2 1,3,2,2,3,5,4,2,4,3
1X10 ⁶	14525.0	617.50	785.80	1,3,4,2,1,1,2,3,3,3 1,3,2,2,3,5,4,3,4,3

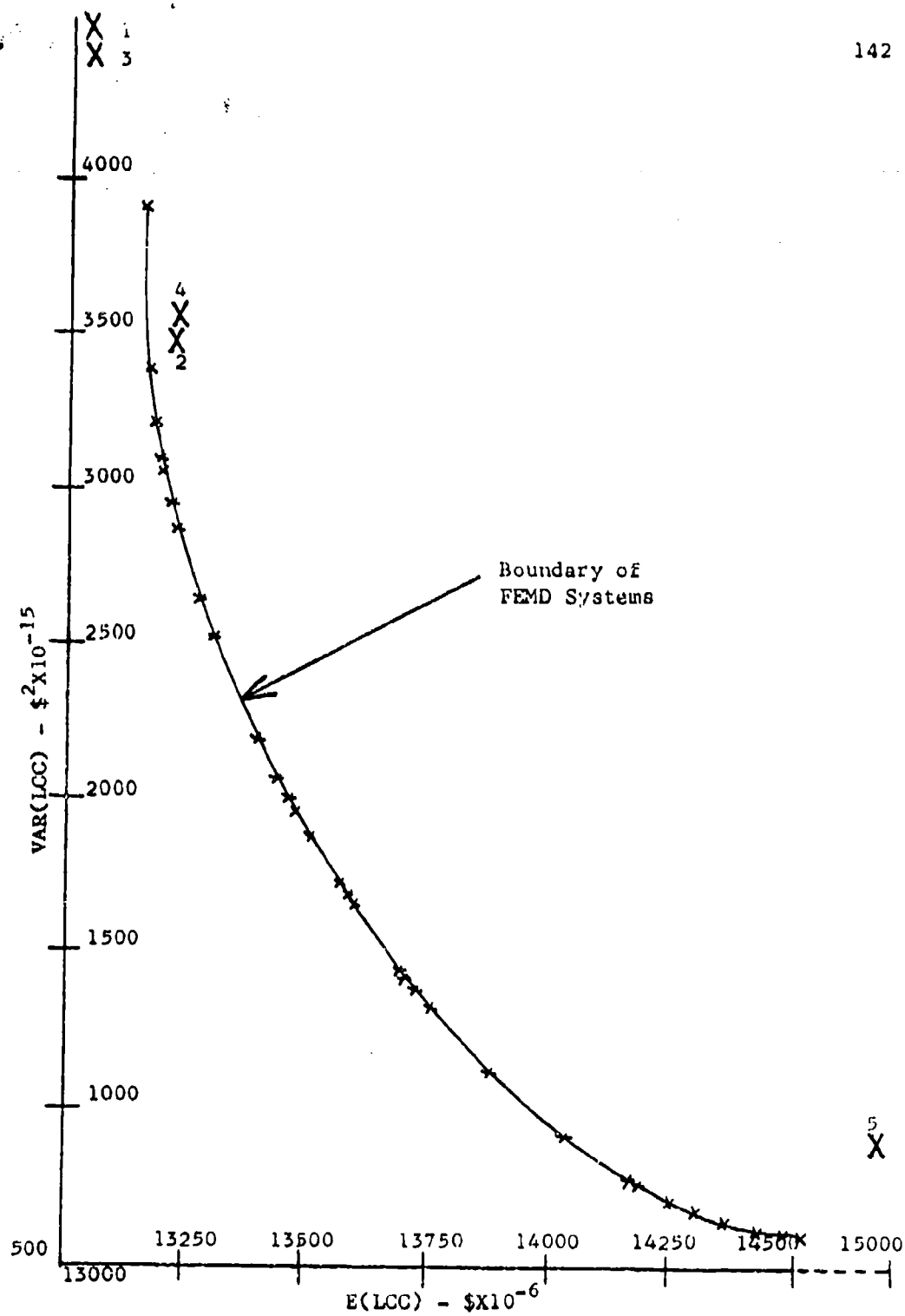


Figure 6.1

FEMD Systems - A and W Chance-Constraints

than our present boundary. Other feasible systems do exist but they lie to the right and above our efficient boundary. These other feasible systems are inefficient in that at least one of the thirty-one systems on the efficient boundary is superior (in an expectation-variance sense) to any feasible system inside the boundary.

Also plotted on the expectation-variance graph shown in Figure 6.1, are points representing the five systems previously generated under risk indifference and deterministic constraints (see Table 6.2). These points are numbered to correspond to the imposed constraint listing of Table 6.2. Examination of the location of these points relative to our efficient boundary reveals that deterministic constraint combinations 1 and 2 (A&W and R, M&W, respectively) provide systems which are infeasible under our availability and weight chance-constraints. On the other hand, deterministic constraint combinations 3 and 4 (A, W & R&D and R, M, W, and R&D) provide systems which are possibly feasible but which are inefficient systems. The last deterministic constraint system (the one with R, M, and W imposed and the objective function changes to minimization of R&D cost) provides a possibly feasible but extremely inefficient system. (The farther a system is from the efficient boundary the more inefficient it is).

Availability, weight, and R&D cost chance-constraints. The effect of imposing an R&D cost chance-constraint in addition to the availability and weight constraints can easily be investigated with

the model. Schedules of efficient systems generated with an R&D cost constraint added are listed in Tables 6.4, 6.5, and 6.6. Three schedules were generated which correspond to R&D cost constraint levels of \$3,500 million, \$3,000 million, and \$2,500 million which are not to be exceeded at a probability level ≥ 0.90 in each case. The efficient boundaries formed by the expectation-variance points corresponding to the systems in these schedules are shown in Figure 6.2. Also shown is the boundary corresponding to the schedule obtained when only availability and weight constraints are imposed. As expected, the number of efficient systems in the schedules is less and the boundaries are smaller when an R&D cost constraint is added. This is caused by the additional R&D cost constraint reducing the size of the set of feasible systems. For the same reason, the smaller the R&D cost constraint level the smaller the boundary. It is also interesting to note that the boundaries which correspond to the schedules generated with an R&D cost constraint added are inefficient (that is, they are higher and to the right of our basic boundary) at the low risk aversion positions (that is, the low expected life cycle cost and high variance position) of the boundaries but coincide with the basic boundary at the high risk aversion positions (that is, the high expected life cycle cost and low variance position) of the boundaries. This would tend to indicate that an R&D cost constraint should be imposed only when the decision-maker is highly averse to risk, since more efficient systems exist without such a constraint when the decision-maker is

Table 6.4

Risk Considered and Availability, Weight
and R&D Cost Chance-Constraints
(R&D \leq \$3500 Million)

Coeff. of Risk Aver. ($\times 10^6$)	E (LCC) (\$ $\times 10^{-6}$)	VAR (LCC) (\$ $\times 10^{-15}$)	STD. DEV. (LCC) (\$ $\times 10^{-6}$)	Subsystem Altern. Selected (SS-1 to 10) (SS-11 to 20)
0	13405.0	2742.75	1656.00	6,8,3,6,3,5,5,8,5,9 2,5,9,8,2,8,7,3,4,2
46	13407.5	2682.75	1637.90	6,8,3,6,2,5,5,8,5,9 3,5,9,8,2,8,7,3,4,2
98	13417.5	2470.25	1571.70	6,4,3,6,8,5,6,8,5,9 2,5,9,7,5,8,7,3,4,2
132	13435.0	2327.75	1525.70	6,4,3,6,2,5,6,8,5,9 3,5,9,7,2,7,7,3,4,3
234	13472.5	2055.25	1433.60	6,4,3,6,2,5,6,8,5,9 3,5,3,7,4,7,7,3,4,3
286	13527.5	1847.50	1359.25	6,5,3,6,5,5,6,3,5,9 3,5,3,5,5,7,7,3,4,3
376	13560.0	1748.75	1322.40	6,5,3,6,8,5,6,8,5,9 3,3,3,5,5,5,7,3,4,3
460	13600.0	1643.75	1282.10	6,5,3,6,2,5,6,3,5,9 3,3,3,5,5,5,7,3,4,3
496	13685.0	1468.75	1211.90	6,5,3,6,8,5,6,3,5,7 3,3,3,5,5,5,7,3,4,3
510	13695.0	1448.75	1203.65	6,5,3,6,2,5,5,3,5,7 3,3,3,5,5,5,7,3,4,3
526	13717.5	1405.0	1185.35	6,5,3,3,8,5,5,3,5,7 3,3,3,5,5,5,7,3,4,3

Table 6.4 (continued)

Coeff. of Risk Aver. ($R \times 10^6$)	E (LCC) ($\$ \times 10^{-6}$)	Var (LCC) ($\$ \times 10^{-15}$)	STD. DEV. (LCC) ($\$ \times 10^{-6}$)	Subsystem Altern. Selected (SS-1 to 10) (SS-11 to 20)
602	13742.5	1359.75	1166.10	3,5,3,3,2,5,5,3,5,7 3,3,3,6,5,5,7,3,4,3
666	13880.0	1141.0	1068.20	3,5,3,3,2,5,5,3,6,7 3,3,3,5,5,5,4,3,4,3
888	14032.5	924.75	961.65	3,5,3,3,2,5,5,3,5,3 3,3,3,5,5,5,4,3,4,3
912	14147.5	797.25	892.90	3,5,3,3,2,5,5,3,5,3 2,3,2,2,3,5,4,3,4,3
1010	14162.5	782.25	884.45	3,5,3,3,2,5,5,3,3,3 3,3,2,2,3,5,4,3,4,3
1146	14220.0	729.75	854.25	3,5,3,3,8,5,3,3,3,3 2,3,2,2,3,5,4,3,4,3
1200	14275.0	683.50	826.75	3,5,3,3,1,5,3,3,3,3 2,3,2,2,3,5,4,3,4,3
1602	14280.0	679.50	824.45	3,5,3,3,1,5,2,3,3,3 2,3,2,2,3,5,4,3,4,3
3936	14355.0	646.25	803.90	3,3,5,3,7,3,2,3,3,3 2,3,2,3,3,4,4,3,4,3
5158	14427.5	631.25	794.50	3,3,4,2,7,3,2,3,3,2 1,3,2,2,3,5,4,3,4,3
5914	14477.5	622.50	789.0	2,3,5,2,1,1,2,3,3,2 1,3,2,2,3,5,4,2,4,3
1×10^6	14525.0	617.50	785.80	1,3,4,2,1,1,2,3,3,3 1,3,2,2,3,5,4,3,4,3

Table 6.5

Risk Considered and Availability, Weight
and R&D Cost Chance-Constraints
(R&D \leq \$3000 Million)

Coeff. of Risk Aver. ($\text{RX}10^6$)	E(LCC) ($\text{\$X}10^{-6}$)	VAR(LCC) ($\text{\$X}10^{-15}$)	STD. DEV. (LCC) ($\text{\$X}10^{-6}$)	Subsystem Altern. Selected (SS-1 to 10) (SS-11 to 20)
0	13710.0	2025.25	1423.10	6,4,3,6,2,5,5,8,5,2 2,5,9,7,2,8,7,3,4,1
68	13717.5	1900.50	1378.60	6,4,3,5,5,5,5,3,5,9 2,2,9,1,2,7,9,3,4,1
90	13732.5	1665.50	1300.20	6,1,3,5,2,5,5,8,5,9 2,2,3,2,2,7,7,3,4,1
192	13747.5	1610.25	1268.95	6,2,3,6,2,5,5,3,5,9 2,2,3,3,1,7,9,3,4,2
272	13755.0	1580.0	1257.0	3,4,3,6,2,5,5,3,5,9 2,2,3,2,1,7,7,3,4,2
286	13777.5	1502.50	1224.85	6,2,3,6,5,5,5,3,5,7 2,5,3,2,2,7,7,3,4,2
300	13845.0	1270.25	1127.55	6,4,3,2,2,5,5,3,5,7 2,3,3,2,2,5,7,3,4,3
526	13862.5	1225.0	1107.95	3,4,3,3,2,5,5,3,5,7 2,3,3,2,2,5,7,3,4,3
544	13905.0	1147.25	1071.10	3,5,3,3,2,5,5,3,5,2 3,3,3,5,5,4,7,3,4,3
562	13955.0	1057.25	1028.25	3,5,3,2,2,5,5,3,5,7 2,3,2,5,3,5,4,3,4,3
674	14032.5	924.75	961.65	3,5,3,3,8,5,5,3,5,3 3,3,3,6,5,5,6,3,4,3
912	14147.5	797.25	892.90	3,5,3,3,2,5,5,3,5,3 2,3,2,2,3,5,4,3,4,3

Table 6.5 (continued)

Coeff. of Risk Aver. ($\times 10^6$)	E (LCC) ($\times 10^6$)	VAR (LCC) ($\times 10^{15}$)	STD. DEV. (LCC) ($\times 10^6$)	Subsystem Altern. Selected (SS-1 to 10) (SS-11 to 20)
1086	14162.5	782.25	884.45	3,5,3,3,5,5,5,3,3,3 3,3,2,2,3,5,4,3,4,3
1146	14220.0	729.75	854.25	3,5,3,3,8,5,3,3,3,3 2,3,2,2,3,5,4,3,4,3
1200	14275.0	683.50	826.75	3,5,3,3,1,5,3,3,3,3 2,3,2,2,3,5,4,3,4,3
1602	14280.0	679.50	824.45	3,5,3,3,1,5,2,3,3,3 2,3,2,2,3,5,4,3,4,3
3936	14355.0	646.25	803.90	3,3,5,3,7,3,2,3,3,3 2,3,2,3,3,4,4,3,4,3
5158	4427.5	631.25	794.50	3,3,4,2,7,3,2,3,3,2 1,3,2,2,3,5,4,3,4,3
5914	14477.5	622.50	789.0	2,3,5,2,1,1,2,3,3,2 1,3,2,2,3,5,4,2,4,3
1×10^6	14525.0	617.50	785.80	1,3,4,2,1,1,2,3,3,3 1,3,2,2,3,5,4,3,4,3

Table 6.6

Risk Considered and Availability, Weight
and R&D Cost Chance-Constraints
(R&D \leq \$2500 Million)

Coeff. of Risk Aver. ($R \times 10^6$)	E (LCC) (\$ $\times 10^{-6}$)	VAR(LCC) (\$ $\times 10^{-15}$)	STD. DEV. (LCC) (\$ $\times 10^{-6}$)	Subsystem Altern. Selected (SS-1 to 10) (SS-11 to 20)
0	14067.5	1434.0	1197.50	3,2,2,5,3,3,5,3,4,2 2,3,9,1,2,7,7,2,4,1
286	14075.0	1128.75	1062.45	3,2,3,2,2,5,5,3,5,2 2,2,3,1,2,7,7,3,4,1
562	14137.5	922.50	960.45	3,4,3,2,2,5,5,3,5,2 2,3,3,2,2,7,4,3,4,2
674	14247.5	741.25	860.95	3,2,5,2,2,5,1,3,5,2 2,3,2,2,3,5,6,3,4,2
3936	14355.0	646.25	803.90	3,3,5,3,7,3,2,3,3,3 2,3,2,3,3,4,4,3,4,3
5158	14427.5	631.25	794.50	3,3,4,2,7,3,2,3,3,2 1,3,2,2,3,5,4,3,4,3
5914	14477.5	622.50	789.0	2,3,5,2,1,1,2,5,3,2 1,3,2,2,3,5,4,3,4,3
1×10^6	14525.0	617.5	785.80	1,3,4,2,1,1,2,3,3,3 1,3,2,2,3,5,4,3,4,3
(R&D \leq \$2250 Millions)				
0	14325.0	845.0	919.25	2,2,2,2,2,2,5,3,5,2 1,3,3,2,2,7,4,2,4,1
1×10^6	14652.5	687.25	829.0	1,5,5,1,1,1,2,3,3,2 1,3,2,2,3,7,4,3,2,2

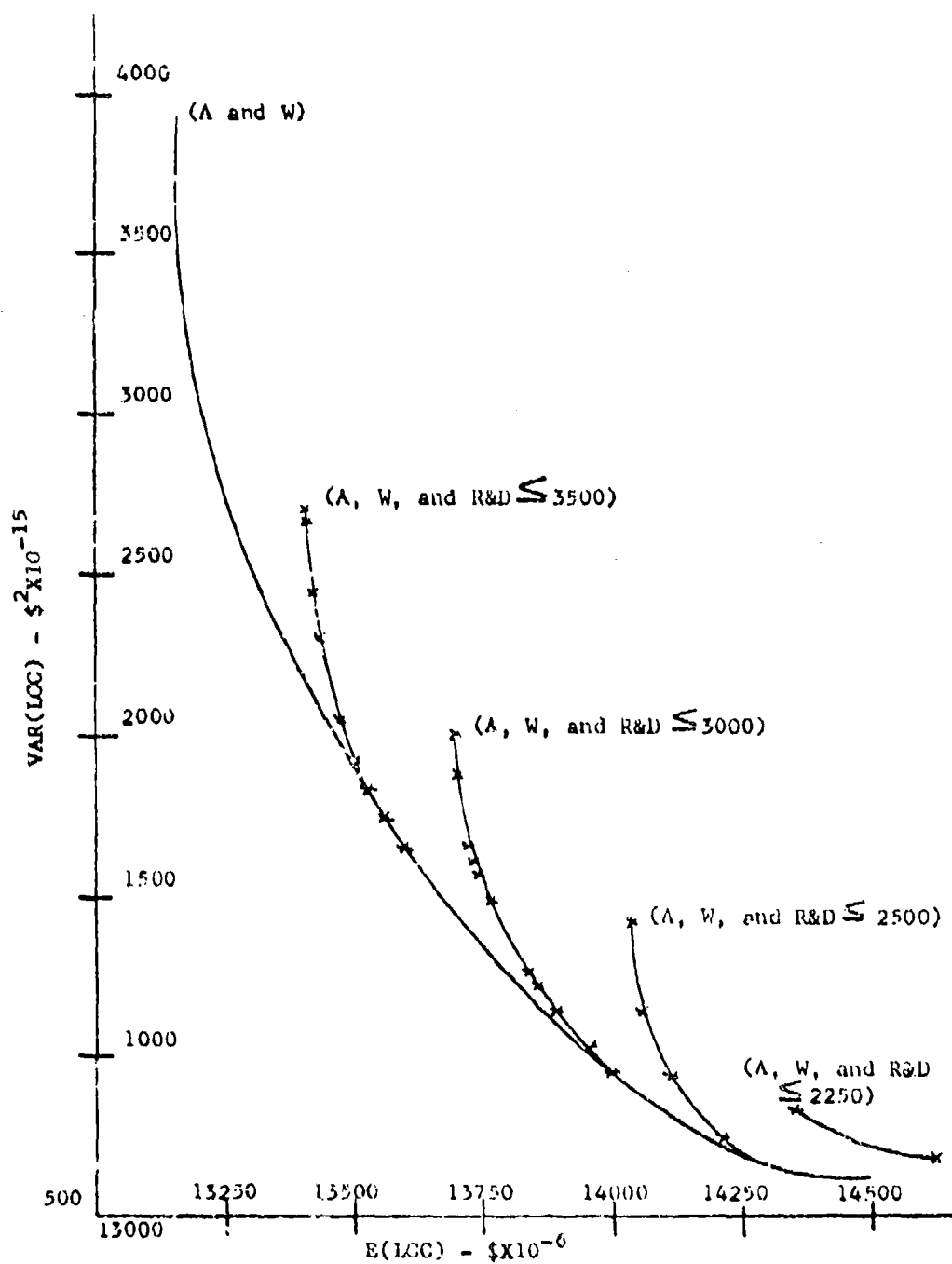


Figure 6.2

F&D Systems - A, W, and R&D Chance-Constraints

willing to tolerate higher risk levels.

Two additional R&D cost constraint levels were also tested with the model. A level of \$2,000 million was found to have no feasible systems possible. And an R&D cost constraints level of \$2,250 million was found to generate the systems listed at the bottom of Table 6.6 for a coefficient of risk aversion of 0.0 and 1.0. The boundary formed by these points is also shown in Figure 6.2. Unlike the other R&D cost boundaries, this boundary does not coincide with the basic boundary at any point but is inefficient over its entire range. Thus, this boundary should be avoided no matter how averse (either high or low) the decision-maker is to risk.

Reliability, maintainability, and weight chance-constraints.

In Chapter II, pages 14-18, we suggested that it is appropriate to view reliability and maintainability combined as a measure of system availability when making decisions pertaining to system cost and operational capability. In generating the previous schedules of efficient systems, we did treat reliability and maintainability together in our availability constraint. Now we treat these support characteristics separately by replacing our availability constraint with separate constraints for reliability and maintainability. Specifically, we require that the system's reliability \leq 8500 failures per month (total force failure rate) and the system's maintainability \leq 3.15 hours (mean time to repair) at a probability level \geq 0.95. These values were chosen so that the system's availability would

continue to be ≥ 0.95 . The same weight constraint is used as that previously imposed (that is, weight ≤ 6000 pounds with probability 0.95 or higher). The schedule of efficient systems generated with these constraints is listed in Table 6.7 and the efficient boundary formed by them is shown in Figure 6.3. Also plotted in this figure for comparative purposes is the basic boundary (that is, the boundary formed by the schedule of efficient systems generated when only availability and weight constraint are imposed).

Examination of this new schedule reveals that it is smaller (21 systems versus 31 in the basic schedule) than the basic schedule and except at low risk aversion levels, inefficient relative to the basic boundary (that is, it is higher and right of our basic boundary). Thus, our suggestion that it is appropriate to view reliability and maintainability combined is supported by these results. The best you can do when you consider reliability and maintainability separately is generate systems equal in efficiency to those generated using an availability constraint. However, this equality degenerates into inefficiency as the risk aversion level increases.

The effect of imposing an R&D cost chance-constraint when reliability and maintainability are treated separately was also investigated. R&D cost constraint levels of \$3,500 million, \$3,000 million and \$2,500 million were imposed at a 0.90 probability level. The boundaries of efficient systems obtained with the R&D cost constraints added are shown in Figure 6.3. The behavior of these

Table 6.7

Risk Considered and Reliability,
Maintainability and Weight
Chance-Constraints

Coeff. of Risk Aver. ($R \times 10^6$)	E LCC ($\$ \times 10^{-6}$)	VAR LCC ($\$ \times 10^{-15}$)	STD. DEV. LCC ($\$ \times 10^{-6}$)	Subsystem Altern. Selected (SS-1 to 10) (SS-11 to 20)
0	13172.5	4127.75	2031.70	6,4,3,9,3,5,9,8,9,9 3,3,5,7,8,9,7,8,8,9
16	13175.0	3441.0	1854.95	6,4,3,6,3,5,9,9,6,9 3,6,5,7,8,9,7,8,7,9
134	13190.0	3070.0	1752.15	6,5,3,6,9,5,6,8,6,9 3,6,5,7,9,8,7,8,7,9
220	13205.0	2965.0	1721.90	6,5,3,6,9,5,6,8,6,9 3,6,5,7,9,7,7,8,7,9
238	13250.0	2770.0	1664.35	6,5,3,6,3,5,6,8,6,9 3,6,5,5,9,7,7,7,7,9
258	13312.5	2515.0	1585.90	6,5,3,6,9,5,6,8,6,9 3,6,3,5,7,7,9,7,7,9
306	13377.5	2462.50	1569.25	6,5,3,6,3,5,6,8,6,9 3,6,3,5,7,7,7,7,7,8
392	13445.0	2081.25	1442.65	6,5,3,6,2,5,5,7,5,9 3,6,5,6,7,7,9,5,4,3
470	13467.5	2031.25	1425.20	6,5,5,6,8,5,6,8,5,9 3,6,5,5,7,5,7,5,4,3
478	13580.0	1794.0	1339.40	6,4,5,6,2,5,5,8,5,7 3,6,3,5,7,5,7,5,4,3
574	13622.5	1709.75	1307.55	3,5,5,6,8,5,5,7,5,7 3,5,5,5,7,5,7,3,4,3

Table 6.7 (continued)

Coeff. of Risk Aver. ($\text{RX}10^6$)	E LCC ($\text{\$X}10^{-6}$)	VAR LCC ($\text{\$X}10^{-15}$)	STD. DEV. LCC ($\text{\$X}10^{-6}$)	Subsystem Altern. Selected (SS-1 to 10) (SS-11 to 20)
680	13810.0	1423.50	1193.10	3,5,5,6,2,5,5,7,5,7 2,3,5,5,7,5,4,5,4,3
880	13850.0	1349.75	1172.50	3,5,5,3,8,5,5,7,5,7 2,3,5,6,7,5,4,5,4,2
1062	13895.0	1331.0	1153.70	3,5,5,2,1,5,5,7,5,7 2,3,5,5,7,5,4,5,4,3
1078	14000.0	1233.50	1110.65	3,5,5,3,2,5,5,8,5,5 2,5,5,2,7,5,4,5,4,1
1116	14055.0	1184.75	1088.45	3,5,5,2,1,5,5,8,5,4 2,5,5,2,7,5,4,5,4,3
1190	14105.0	1141.0	1068.20	3,5,5,2,1,5,5,8,5,4 2,3,5,2,7,4,4,5,4,4
1456	14165.0	1092.25	1045.10	3,5,5,2,7,5,2,8,5,5 1,3,5,3,7,4,6,5,4,4
1760	14277.5	1027.50	1013.65	3,4,4,2,4,4,2,8,5,5 1,3,5,2,7,5,4,5,4,1
2474	14280.0	1021.25	1010.55	2,5,5,5,1,4,2,5,5,4 1,3,4,2,7,5,4,5,4,4
$1\text{X}10^6$	14400.0	997.25	998.60	2,5,4,4,1,4,2,8,4,4 1,3,2,2,7,5,4,5,4,1

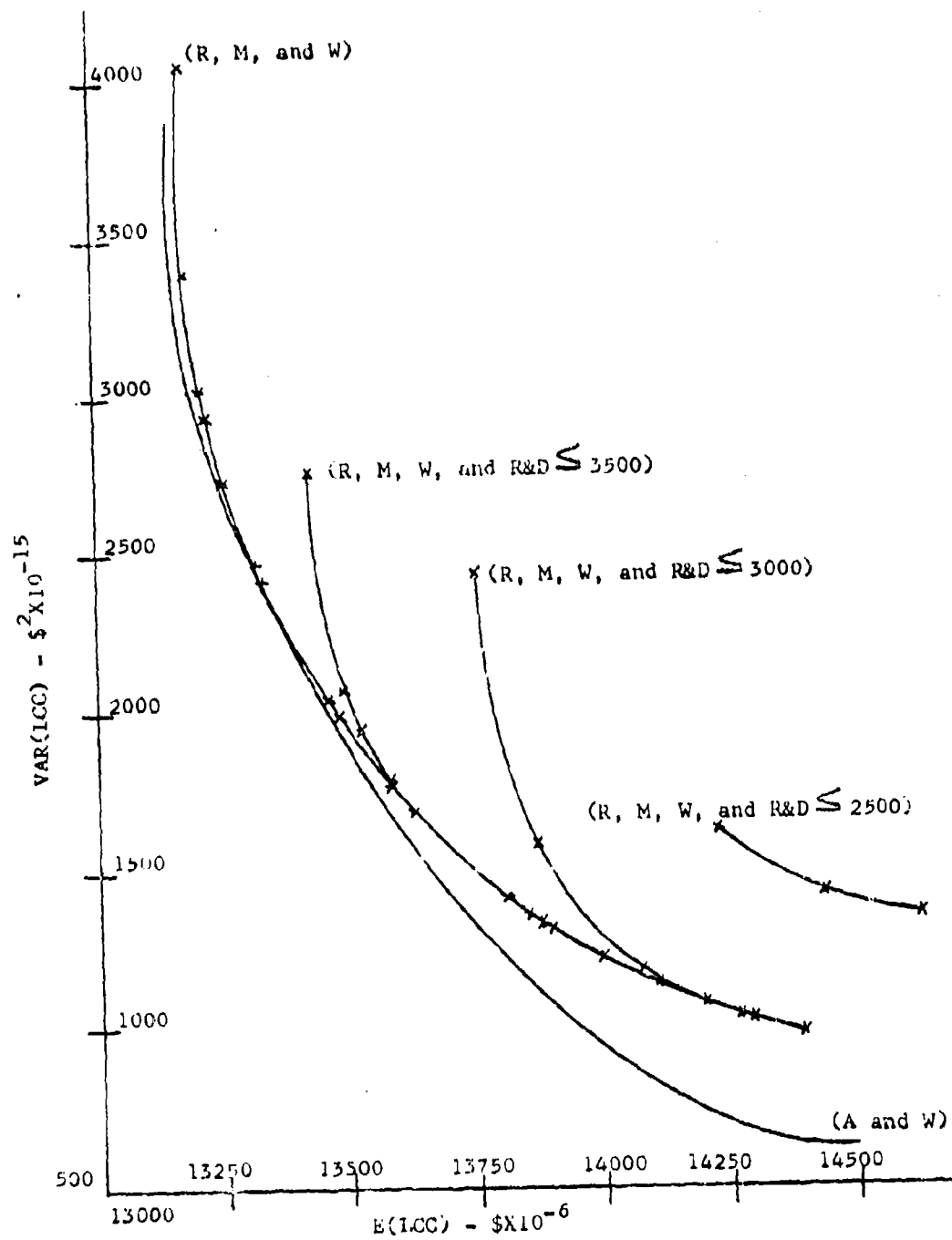


Figure 6.3

FEND Systems - R, M, and W and R, M, W,
and R&D Chance-Constraints

boundaries relative to the boundary obtained when reliability and maintainability were treated separately without an R&D cost constraint is quite similar to that previously observed when R&D cost constraints were added to our basic constraints (Figure 6.2). The boundaries under R&D cost constraints are inefficient to the one generated when such constraints are not imposed. The inefficiency being the highest at low risk aversion levels and increasing as the R&D constraint level is reduced.

Final System Selection

In the previous section we demonstrated how the model and solution technique may be used to generate schedules of efficient systems. We also showed how the effect of imposing various constraint combinations could be examined by comparing the various schedules generated. Now we turn our attention to the problem of selecting a final system from a schedule of efficient systems.

As demonstrated in the previous section, the model and solution technique reduce an unmanageable number of system combinations (for our example problems there were $(9)^{20}$ possible system combinations) to a relatively small set of efficient systems which should be presented to the decision-maker who then selects what he regards as the preferred system from this greatly reduced set of efficient systems. To facilitate this final selection, it would be helpful if we could provide the decision-maker with more information than that provided by estimates of means and variances for the life cycle

cost of the systems in the efficient schedule. The amount of risk implied by these estimates is not readily apparent to the typical decision-maker. Thus, it would be useful to derive additional information which would aid the decision-maker in assessing the risk associated with the various efficient systems.

Non-parametric analysis. Before investigating what information we can derive from the probability distribution describing a system's life cycle cost, it would be well to determine what conclusions can be drawn, if any, without any knowledge of (or assumptions about) the distribution other than its expected value and variance.

To assist the decision-maker to understand the meaning and significance of expected values and standard deviations (variances) several types of calculations could be made. One such type would concern upper bounds on the amount of risk involved. The well-known Tchebycheff inequality yields

$$\text{PR} \left[\left| \text{LCC} - E[\text{LCC}] \right| \geq k \sqrt{\text{Var}[\text{LCC}]} \right] \leq k^{-2}$$

for all k regardless of the distribution of LCC. Thus, for example, the probability is no more than 0.25, 0.04, and 0.01, respectively, that LCC will be greater than $E[\text{LCC}] + 2\sqrt{\text{Var}[\text{LCC}]}$, $E[\text{LCC}] + 5\sqrt{\text{Var}[\text{LCC}]}$, and $E[\text{LCC}] + 10\sqrt{\text{Var}[\text{LCC}]}$, respectively. However, the Tchebycheff inequality usually is very conservative. For most distributions, it is "very unlikely" that a given observation will lie above the mean plus three standard deviations. Therefore,

the risk associated with LCC for a particular system may be partially described by using the Tchebycheff's inequality, but these results should be tempered by comparison with a calculations for a distribution, such as the normal distribution, where $E[LCC] + 3 \sqrt{\text{Var}[LCC]}$ may be considered as almost an upper bound on LCC. More precise statements about a system's "risk" require knowledge of the functional form of the distribution of LCC for each efficient system.

Approximate distribution of cost. In our model, the actual underlying distribution of the LCC_{ij} for each subsystem are unknown. Therefore, the distribution of their sum, LCC, is also unknown. Thus, we must assume a functional form for LCC if we wish to make more precise statements about the amount of risk associated with the efficient systems in our schedule. Using the same rational (Chapter IV, pages 80-81) we used for assuming approximate normality for the sum of our external constraint values, it appears reasonable to assume that LCC would also be approximately normal. Since our subsystem values are independent, this normality assumption requires only that no individual LCC_{ij} 's have such large variances relative to the others that their distribution dominate the distribution of the sum.

If the subsystem life cycle cost data for the system being studied are such that the normality assumption appears reasonable, then we could provide the decision-maker with considerably more information than is possible using only the mean and variance estimates.

For instance, we could aid the decision-maker in obtaining a "feel" for the riskiness of each system by setting up a table similar to Table 6.8 which indicates for each efficient system (the schedule used for this table is that generated when only availability and weight chance-constraints were imposed (see Table 6.3)) the probability of obtaining various size system life cycle cost.

The probability values in this table were computed as follows: if system 1 ($E[LCC_1] = 13,162.5$ and $Var[LCC_1] = 3,954,000$) is selected, the probability of incurring a cost greater than \$14,000 million (under our normality assumption) is .5 - the probability corresponding to the standard normal deviate

$$Z = \frac{14,000 - 13,162.5}{\sqrt{3,954,000}} = 0.42.$$

From standard normal tables, $Pr(LCC > \$14,000 \text{ million}) = 0.5 - .1628 = 0.3372$.

The particular levels of system life cycle cost used in setting the table would be stipulated by the decision-maker. Such information, when presented to the decision-maker, should provide substantial insight into the "risk" associated with each of the efficient systems. For instance, the decision-maker could then use some form of aspiration level principle (for example, he could select the efficient system which minimizes the probability of exceeding the aspiration level cost) to make his final selection.

Table 6.8

System Risk Information For
Final System Selection

Sys. No. ³⁶	Pr(LCC > \$14B)	Pr(LCC > \$15B)	Pr(LCC > \$16B)	Pr(LCC > \$17B)
1	.3372	.1788	.0778	.0274
2	.3264	.1611	.0630	.0192
3	.3228	.1562	.0594	.0174
4	.3112	.1515	.0559	.0158
5	.3112	.1515	.0548	.0150
6	.3112	.1469	.0515	.0135
7	.3228	.1469	.0505	.0129
8	.3264	.1446	.0475	.0113
9	.3336	.1446	.0451	.0102
10	.3409	.1401	.0401	.0078
11	.3483	.1401	.0384	.0069
12	.3483	.1379	.0367	.0064
13	.3520	.1379	.0359	.0060
14	.3594	.1379	.0344	.0054
15	.3783	.1379	.0307	.0040
16	.3783	.1379	.0307	.0040
17	.3974	.1379	.0274	.0030
18	.3974	.1401	.0274	.0030
19	.4052	.1401	.0272	.0029
20	.4129	.1401	.0262	.0026

Table 6.8 (continued)

Sys. No.	Pr(LCC > \$14B)	Pr(LCC > \$15B)	Pr(LCC > \$16B)	Pr(LCC > \$17B)
21	.4522	.1469	.0239	.0018
22	.5359	.1711	.0233	.0012
23	.5675	.1711	.0192	.0007
24	.5714	.1711	.0188	.0007
25	.6064	.1814	.0179	.0005
26	.6255	.1894	.0183	.0005
27	.6368	.1921	.0183	.0005
28	.6700	.2090	.0197	.0005
29	.7088	.2358	.0233	.0006
30	.7291	.2546	.0268	.0007
31	.7454	.2709	.0300	.0008

³⁶ System numbers are keyed to order of efficient system listing as listed in Table 6.3.

Computation Time Statistics

The generation of a single efficient system requires solution of a large integer programming problem, and generation of a schedule of efficient systems requires numerous such solutions. Therefore, if our model is to be of practical use, our solution algorithm must provide solutions fairly rapidly. The following time statistics accumulated from our example problem runs indicate that the computation time required to obtain solutions is not excessive, and therefore the model is readily suitable for reliability/maintainability analysis of large multi-subsystem systems:

Table 6.9

Example Problem
Time Statistics

Constraints Imposed	No. of Solutions Req'd for Generating Schedule of efficient Systems	Aver. Time/Solution (Minutes)
A & W	57	0.663
A, W, and R&D		
(R&D \leq 3500)	21	1.385
(R&D \leq 3000)	23	6.510
(R&D \leq 2500)	8	2.918
FR, MTTR, and W	41	4.381

CHAPTER VII

SUMMARY AND RECOMMENDATIONS

This study combines aspects of utility theory, probability theory, and mathematical programming to develop an efficient method for performing reliability/maintainability analysis during the early conceptual stages of system development. The general concepts of operational capability and system life cycle cost are used to construct an operational model for determining subsystem reliability and maintainability "design point" characteristics. Consideration of both technological and cost uncertainty is incorporated within the model. Specifically, the method developed provides a systematic and operationally efficient technique for selecting subsystem reliability/maintainability alternatives when (1) attainable subsystem reliability/maintainability levels are not known with certainty, (2) all life cycle element cost flows are not known with certainty, (3) chance-constraint restrictions exist on some or all of the following factors: system availability, weight, reliability, maintainability, and/or R&D cost, (4) some subsystem reliability/maintainability design alternatives are interdependent (contingent and/or mutually-exclusive) (5) the suitability of selecting any particular system (combination of subsystem reliability/maintainability alternatives) depends upon both cost and risk considerations.

An efficient solution algorithm was developed for solving the model by modifying a zero-one linear programming algorithm developed by Geoffrion to capitalize on the multiple-choice structure of the problem. Lastly, the utility of the model and solution technique were demonstrated in example problems involving a large multi-subsystem system.

Numerous opportunities exist for additional research based on the concepts developed in this dissertation. Some of the areas which especially warrant further investigation include the following:

1. The utility of the model needs to be tested in an actual development situation. Implementation of the model in an actual development environment would provide an evaluation of the usefulness of the model and the information needed for enhancing the utility of the model.

2. Adaptation of the model to fit other later development stages should be investigated. Reliability/maintainability analysis is a continuing requirement throughout the acquisition process. Tradeoffs must be made during all stages of system development not just during the conceptual phase.

3. Possible expansion of the model to consider performance characteristics as variable factors rather than given factors should be studied. This would require development of methods for determining how changes in performance characteristics would affect

the available reliability/maintainability alternatives.

4. The use of other criterion functions than the one used in the present model should be investigated. The expectation-variance utility function used in the model is only an approximation to a true utility function. Other approximations have been and can be developed and their use as the model's criterion function should be examined.

5. Use of the "multiple-choice" zero-one linear programming algorithm for problems in other areas should be investigated. The literature contains examples of problems from several other areas which seem to fit the multiple-choice structure of the model developed in this dissertation.

6. Use of the model output for establishing system contract incentives appears reasonable. Incentives on such items as system reliability, maintainability, and weight, could be analyzed using the output information obtained in solving the model.

APPENDIX A

MULTIPLE-CHOICE SOLUTION
ALGORITHM COMPUTER PROGRAM

12-18-71 16.598 RAM DECISION MODEL

MAIN PROGRAM

```

C          RAM DECISION MODEL
C          MAIN PROGRAM
      DIMENSION A(40,180),H(180),C(180),XX(180),Y(180),T(1
•80),CON(10),PA(5,180),PB(10),PELCC(180),PVLCC(180),R
•ISK(25),PEB(5,180),PVB(5,180),TT(180),IT(180),ZSTD(1
•0)
      INTEGER I
      DOUBLE PRECISION XX,Y,PEB,PVB,TEMP,VARTOT,TT,PA,ZSTD
•,STD,Z1,Z2,Z3,Z4,Z10I,PB,CON
      NRUN = 0
      READ 10,ISS,N,M,ICON,ICON1,IRISK,IVER,IDEF
10  FORMAT(B13)
      READ 20,(ZSTD(I),I=1,ICON)
      READ 20,(CON(J),J=1,ICON)
20  FORMAT(6F12.4)
      READ 30,(RISK(J),J=1,IRISK)
30  FORMAT(6F12.4)
40  J1 = -2
      J2 = 0
      LL = N/3
      DO 55 I=1,LL
        J1 = J1 + 3
        J2 = J2 + 3
        READ 50,(PELCC(J),PVLCC(J),J=J1,J2)
50  FORMAT(6F12.4)
55  CONTINUE
      DO 120 II=1,ICON
        Z1 = 0.0
        Z2 = 0.0
        Z3 = 0.0
        VARTOT = 0.0
        IF(II.EQ.5)GO TO 66
        J1 = -2
        J2 = 0
        DO 69 I=1,LL
          J1 = J1 + 3
          J2 = J2 + 3
          READ 50,(XX(J),Y(J),J=J1,J2)
60  CONTINUE
          DO 65 I=1,N
            PEB(II,I) = XX(I)
            PVB(II,I) = Y(I)
65  CONTINUE
            IF(II.NE.4)GO TO 66
            DO 67 I=1,N
              PEB(4,I) = PEB(4,I)/60.
              PVB(4,I) = PVB(4,I)/3600.
              XX(I) = PEB(2,I)*(PEB(4,I)-CON(4))
              Y(I) = (PVB(2,I)*(PEB(2,I)*2))*(PVB(4,I)*((PEB(4,I)
•-CON(4))*2)-(PEB(2,I)*2)*((PEB(4,I)-CON(4))*2)

```

12-18-71 16.596 RAM DECISION MODEL

MAIN PROGRAM

```

PEB(5,1) = PEB(4,1)
PVB(5,1) = PVB(4,1)
67 CONTINUE
66 IF(11.NE.5)GO TO 68
DO 69 I=1,N
XX(1) = 1. - (PEB(2,1)*PEB(4,1))/(700.*720.)
Y(1) = (((PEB(2,1)**2)*PVB(4,1)) + ((PEB(4,1)**2)*PV
*B(2,1)) + (PVB(2,1)*PVB(4,1)))/(700.*720.)
Y(1) = Y(1)/(700.*720.)
PEB(4,1) = XX(1)
PVB(4,1) = Y(1)
TEMP = XX(1)
XX(1) = DLOG(XX(1)) - Y(1)/(2.*(XX(1)**2))
Y(1) = Y(1)/(TEMP**2)
69 CONTINUE
68 DO 70 I=1,N
VARTOT = VARTOT + Y(1)
70 CONTINUE
71 DO 80 I=1,N
TT(1) = DSORT(VARTOT) - DSORT(VARTOT - Y(1))
72 Z1 = Z1 + TT(1)
IF(11.NE.5)GO TO 73
PA(11,1) = XX(1) + TT(1)*ZSTD(11)
GO TO 80
73 PA(11,1) = -(XX(1) + TT(1)*ZSTD(11))
80 CONTINUE
SIDT = DSORT(VARTOT)
NN = N - 1
DO 100 I=1,NN
IJ = I + 1
DO 100 J=IJ,N
IF(Y(I) - Y(J))100,100,90
90 TEMP = Y(I)
Y(I) = Y(J)
Y(J) = TEMP
100 CONTINUE
KK = N - 155
DO 110 I=1,KK
Z2 = Z2 + SIDT - DSORT(VARTOT - Y(1))
Z3 = Z3 + Y(1)
110 CONTINUE
Z4 = DSORT(VARTOT - Z3)
ZTOT = ZSTD(11)*(Z2 + Z4 - Z1)
IF(11.NE.4)GO TO 111
PB(11) = -ZTOT
GO TO 120
111 IF(11.NE.5)GO TO 112
PB(11) = -DLOG(CON(11)) + ZTOT
GO TO 120
112 PB(11) = CON(11) - ZTOT

```

12-18-71 16.598 RAM DECISION MODEL

MAIN PROGRAM

```

120 CONTINUE
  DO 125 J=1,N
    TEMP = PA(4,J)
    PA(4,J) = PA(5,J)*100000.
    PA(5,J) = TEMP
125 CONTINUE
  TEMP = PB(4)
  PB(4) = PB(5)*100000.
  PB(5) = TEMP
  GO TO (126,127,128,129,130,131), IVER
126 DO 132 I=1,ICON
  IF(1.EQ.1.OR.1.EQ.2.OR.1.EQ.5)GO TO 132
  I1 = I - 2
  DO 133 J=1,N
  A(I1,J) = PA(1,J)
133 CONTINUE
  B(I1) = PB(1)
132 CONTINUE
  ICON = ICON - 3
  GO TO 143
127 DO 134 I=1,ICON
  IF(1.EQ.2.OR.1.EQ.5)GO TO 134
  I1 = I
  IF(1.EQ.3.OR.1.EQ.4)I1=I - 1
  DO 135 J=1,N
  A(I1,J) = PA(1,J)
135 CONTINUE
  B(I1) = PB(1)
134 CONTINUE
  ICON = ICON - 2
  GO TO 143
128 DO 136 I=1,ICON
  IF(1.EQ.1)GO TO 136
  IF(1.EQ.1.OR.1.EQ.4)GO TO 136
  I1 = I - 1
  IF(1.EQ.5)I1=I1-1
  DO 137 J=1,N
  A(I1,J) = PA(1,J)
137 CONTINUE
  B(I1) = PB(1)
136 CONTINUE
  ICON = ICON - 2
  GO TO 143
129 DO 138 I=1,ICON
  IF(1.EQ.4)GO TO 138
  I1 = I
  IF(1.EQ.5)I1=I1-1
  DO 139 J=1,N
  A(I1,J) = PA(1,J)
139 CONTINUE

```

12-18-71 16.598 RAM DECISION MODEL

MAIN PROGRAM

```

      B(11) = PB(1)
138  CONTINUE
      ICON = ICON - 1
      GO TO 143
139  DO 140 I=1,ICON
      DO 141 J=1,N
      A(1,J) = PA(1,J)
141  CONTINUE
      B(1) = PB(1)
140  CONTINUE
      GO TO 143
131  DO 142 I=1,ICON
      IF(1.EQ.1.OR.1.EQ.4)GO TO 142
      IF(1.EQ.2.OR.1.EQ.3)I1=I-1
      IF(1.EQ.5)I1 = I-2
      DO 144 J=1,N
      A(I1,J) = PA(I,J)
144  CONTINUE
      B(11) = PB(1)
142  CONTINUE
      ICON = ICON - 2
143  IF(IDEP.NE.1)GO TO153
C*****CONTINGENT AND EXCLUSIVE CONSTRAINTS EXIST*****
C*****HIGH R LEVELS FOR SS 18,19,20 CONTINGENT*****
C*****HIGH R LEVELS FOR SS 13,15,16 MUTUALLY EXCLUSIVE*****
      I1 = ICON
      ICON = ICON + 3
      ICON1 = ICON1 + 1
      I2 = ICON1 + 1
      I3 = ICON1 + 2
      DO 700 I=1,N
      A(ICON,I) = A(I1,I)
700  CONTINUE
      B(ICON) = B(I1)
      DO 710 I=1,N
      A(ICON1,I) = 0.0
      A(I2,I) = 0.0
      A(I3,I) = 0.0
710  CONTINUE
      DO 720 I=111,117,3
      A(ICON1,I) = -1.0
720  CONTINUE
      DO 730 I=129,144,3
      A(ICON1,I) = -1.0
730  CONTINUE
      DO 740 I=160,162
      A(I2,I) = 2.0
      A(I3,I) = -2.0
740  CONTINUE
      DO 750 I=169,171

```

12-18-71 16.598 RAM DECISION MODEL

MAIN PROGRAM

```

      A(12,1) = -1.0
      A(13,1) = 1.0
750  CONTINUE
      DO 740 I=178,180
      A(12,1) = -1.0
      A(13,1) = 1.0
760  CONTINUE
      B(ICON1) = 1.0
      B(12) = 0.0
      B(13) = 0.0
153  MM = -1
      NSS = N/ISS
      I3 = ICON + 1
      I1 = I3 + 1
      I2 = I1 + (ISS - 1)
      DO 150 I=I1,I2
      B(I) = 1.
      MM = MM + 1
      DO 150 J=1,NSS
      JJ = J + MM*NSS
      A(1,JJ) = -1.
150  CONTINUE
      B(13) = -ISS
      DO 160 I=1,N
      A(13,I) = 1.0
160  CONTINUE
      I NRUN = NRUN + 1
      DO 170 I=1,N
      T(I) = 0
      IF(IVER.NE.6)GO TO 161
      C(I) = PEB(1,I) + RISK(NRUN)*PVB(1,I)
      GO TO 170
161  C(I) = PELCC(I) + RISK(NRUN)*PVLCC(I)
170  CONTINUE
500  CALL RAM(A,B,C,T,NRUN,ICON,ICON1,RISK)
      ELCC = 0.0
      VLCC = 0.0
      ERDC = 0.0
      VRDC = 0.0
      EFR = 0.0
      VFR = 0.0
      EW = 0.0
      VW = 0.0
      EMTIR = 0.0
      VMTIR = 0.0
      EAV = 1.0
      VAV1 = 1.0
      VAV2 = 1.0
      DO 165 I=1,N
      IF(T(I).EQ.0)GO TO 165

```

12-18-71 16.598 RAM DECISION MODEL

MAIN PROGRAM

```

ELCC = ELCC + PELCC(1)
VLCC = VLCC + PVLCC(1)
ERUC = ERUC + PER(1,1)
VRUC = VRUC + PVR(1,1)
EFR = EFR + PER(2,1)
VFR = VFR + PVR(2,1)
EW = EW + PER(3,1)
VW = VW + PVR(3,1)
XX(1) = PER(2,1)*PER(5,1)
Y(1) = (PVB(2,1)+(PER(2,1)**2))*(PVB(5,1)*(PER(5,1)*
*2))-((PER(2,1)**2)*(PER(5,1)**2))
EAV = EAV*PER(4,1)
VAV1 = VAV1*(PVB(4,1) + (PER(4,1)**2))
VAV2 = VAV2*(PER(4,1)**2)
165 CONTINUE
VAV = VAV1 - VAV2
DO 176 I=1,N
IF(IT(1).EQ.0)GO TO 176
EMTIR = EMTIR + XX(1)/EFR
VMTIR = VMTIR + (Y(1)+XX(1)**2)/(VFR+EFR**2)-((XX(1)
**2)/(EFR**2))
176 CONTINUE
EMTIR = EMTIR*60.
VMTIR = VMTIR*3600.
DO 169 I=1,N
IT(1) = T(1)
169 CONTINUE
NN = N - 1
DO 167 I=1,NN
IF(IT(1).EQ.0)GO TO 167
IJ = I + 1
DO 166 J=IJ,N
IF(IT(1).LE.IT(J))GO TO 166
ITEMP = IT(1)
IT(1) = IT(J)
IT(J) = ITEMP
166 CONTINUE
167 CONTINUE
KSS = N/ISS
K = 0
DO 168 I=1,N
IF(IT(1).EQ.0)GO TO 168
K = K + 1
II(K) = IT(1) - ((IT(1) - 1)/KSS)*KSS
168 CONTINUE
PRINT 9993
9993 FORMAT(1H1)
PRINT 199,NRUN
199 FORMAT(/////JUX,23HRESULTS FOR RISK LEVEL ,12//)
PRINT200,RISK(NRUN)

```

12-18-71 16,598 RAM DECISION MODEL

MAIN PROGRAM

```

200 FORMAT(19X,22H COEFFICIENT OF RISK =,F10.7)
    IF(IVLR.EQ.1)GO TO 601
    IF(IVER.EQ.3)GO TO 609
    IF(IVER.EQ.6)GO TO 600
    PRINT 201,CUN(1)
201 FORMAT(19X,25H BUDGETED R AND D FUNDS =,F7.1)
    IF(IVER.EQ.2)GO TO 601
600 PRINT 202,CUN(2)
202 FORMAT(19X,23H MAXIMUM FAILURE RATE =,F7.1)
601 PRINT 203,CUN(3)
203 FORMAT(19X,17H MAXIMUM WEIGHT =,F7.1)
    IF(IVER.EQ.1)GO TO 602
    IF(IVER.EQ.2)GO TO 602
    RMTTR = CUN(4)*60.
    PRINT 204,RMTTR
204 FORMAT(19X,30H MAXIMUM MEAN TIME TO REPAIR =,F6.1)
    IF(IVER.EQ.3)GO TO 206
    IF(IVER.EQ.4)GO TO 206
    IF(IVER.EQ.6)GO TO 206
602 PRINT 205,CUN(5)
205 FORMAT(19X,33H REQUIRED INHERENT AVAILABILITY =,F7.4
    .)
206 PRINT 9992
9992 FORMAT(1H0)
    PRINT 300
300 FORMAT(19X,31H THE OPTIMAL SYSTEM RESULTS ARE,/)
    PRINT 301,ELCC
301 FORMAT(19X,28H EXPECTED LIFE CYCLE COSTS =,F8.1)
    PRINT 302,VLCC
302 FORMAT(19X,31H VARIANCE OF LIFE CYCLE COSTS =,F11.1)
    SIDLCC = SORT(VLCC)
    PRINT 309,SIDLCC
309 FORMAT(19X,38H STANDARD DEVIATION OF LIFE CYCLE COST
    .,JHS =,F7.1)
    PRINT 9992
    PRINT 303,ERDC
303 FORMAT(19X,25H EXPECTED R AND D COSTS =,F7.1)
    PRINT 304,VRDC
304 FORMAT(19X,28H VARIANCE OF R AND D COSTS =,F8.1)
    STORDC = SORT(VRDC)
    PRINT 501,STORDC
501 FORMAT(19X,38H STANDARD DEVIATION OF R AND D COSTS =
    .,F6.1)
    PRINT 9992
    PRINT 305,EFR
305 FORMAT(19X,24H EXPECTED FAILURE RATE =,F7.1)
    PRINT 306,VFR
306 FORMAT(19X,27H VARIANCE OF FAILURE RATE =,F9.1)
    STDFR = SORT(VFR)
    PRINT 502,STDFR

```

12-18-71 16.59H RAM DECISION MODEL

MAIN PROGRAM

```

502 FORMAT(19X,37H STANDARD DEVIATION OF FAILURE RATE =,
  *F6.1)
  PRINT 9992
  PRINT 307,EH
307 FORMAT(19X,18H EXPECTED HEIGHT =,F7.1)
  PRINT 308,VH
308 FORMAT(19X,21H VARIANCE OF HEIGHT =,F8.1)
  STDH = SQRT(VH)
  PRINT 503,STDH
503 FORMAT(19X,31H STANDARD DEVIATION OF HEIGHT =,F6.1)
  PRINT 9992
  PRINT 504,EMTR
504 FORMAT(19X,31H EXPECTED MEAN TIME TO REPAIR =,F6.1)
  PRINT 505,VMTR
505 FORMAT(19X,34H VARIANCE OF MEAN TIME TO REPAIR =,F6.
  *1)
  STDVTR = SQRT(VMTR)
  PRINT 506,STDVTR
506 FORMAT(19X,29H STANDARD DEVIATION OF MTR =,F5.1)
  PRINT 9992
  PRINT 507,EAV
507 FORMAT(19X,24H EXPECTED AVAILABILITY =,F7.4)
  PRINT 508,VAV
508 FORMAT(19X,27H VARIANCE OF AVAILABILITY =,F11.8)
  STDAV = SQRT(VAV)
  PRINT 509,STDAV
509 FORMAT(19X,37H STANDARD DEVIATION OF AVAILABILITY =,
  *F7.4)
  PRINT 9993
  PRINT 400
400 FORMAT(//////////25X,26H THE OPTIMAL SUBSYSTEM ALL
  *12LOCATIONS ARE,/)
  DO 402 I=1,ISS
  PRINT 401,I,IT(I)
401 FORMAT(30X,11H SUBSYSTEM ,12,3X,10HLEVEL IS =,I2)
402 CONTINUE
  IF(NRUN - IRISK)1,100,180
180 CALL EXIT
  END

```

6 WORDS OF MEMORY USED BY THIS COMPILATION

12-18-71 16.614 RAM DECISION MODEL

SUBROUTINE RAM

```

C      RAM DECISION MODEL
C      SUBROUTINE RAM
SUBROUTINE RAM(A,U,C,I,NRUN,ICON,ICON1,RISK)
  DIMENSION CMIN(20),AMAX(20),RISK(25)
  DIMENSION A(40,180),XL(180),IXINS(180)
  DIMENSION B(180),C(180),BS(180),S(180),SH(180),NS(18
  *0)
  DIMENSION SMAX(180),SMAXB(180),T(180),CS(180),D(180)
  DIMENSION JH(180),XX(180),Y(180),KO(6),JSS(20)
  INTEGER S,SMAX,SC,T
  COMMON HS(180),ZBAR,LPSEQ
  DATA HC18/6HB /
  DATA HLAN/6H /
  DO 99 I=1,20
    JSS(I) = 0
    CMIN(I) = 0.0
    AMAX(I) = 0.0
  99 CONTINUE
100 DO 110 I=1,180
    XL(I) = 0.0
    D(I) = 0.0
    JH(I) = 0.0
    XX(I) = 0.0
    Y(I) = 0.0
    OBJ = 0.0
    IXINS(I) = 0
    BS(I)=0.0
    S(I)=0
    SH(I)=BLANK
    NS(I)=0
    SMAX(I)=0
    SMAXB(I)=BLANK
110 CONTINUE
    DO 255 J=1,180
      CS(J)=C(J)
255 CONTINUE
    LPSEQ = 0
    II=0
    NOPT=0
    NFATH=0
    NSIMP=0
    IPOST=1
    IFRST=1
C      MINIMIZE SUM C(J)*X(J)
C      CONSTRAINTS ARE B(I)+SUM A(I,J)*X(J) GE ZERO
    READ 9000,M,N,ISS,L,SC,ZBAR,ISCHMAX,ISCFR,NOP,ZKBAR,H
    *1,H2
9000 FORMAT(5I3,F12.4,3I3,F12.4,2A6)
    PRINT 9993
    PRINT 9001,M,N,ISS,L,SC,ZBAR,ISCHMAX,ISCFR,NOP,ZKBAR,

```

12-18-71 16.614 RAM DECISION MODEL

SUBROUTINE RAM

```

      *H1,H2
9001 FORMAT (D13.1X,F12.4,3I3,F12.4,1X,2A6)
      MU=M
      M1=M0+1
      KLS = N/135
      JSCFR=JSCFR
      ZBAR=ZBAR*.99999
      PRINT 9010,M,N
9010 FORMAT (JH0M=,13,2X,2HN=,13)
      PRINT 9992
9991 FORMAT (1H )
9992 FORMAT (1H0)
9993 FORMAT (1H1)
9888 IF (L.GE.0) GO TO 130
      L=0
      IFRST=0
130 CONTINUE
      PRINT 9992
      PRINT 9599,(C(J),J=1,N)
9599 FORMAT(9F13.3)
      IF (NOP.NE.1) GO TO 252
      PRINT 9992
      PRINT 9500,(B(I),I=1,M)
      PRINT 9992
      DO 251 I=1,ICON
      PRINT 9600,(A(I,J),J=1,N)
      PRINT 9991
251 CONTINUE
2600 FORMAT(4(1X,F12.4))
252 CONTINUE
      IF (ZBAR.GT.0.0) GO TO 300
      ZBAR=0.0
      DO 275 I=1,N
275 ZBAR=ZBAR+C(J)
300 ZS=J.0
      DO 325 I=1,M
325 BS(I)=B(I)
      DO 330 J=1,N
330 N(J)=J
      IF (MU+ISCMAX.GT.40) ISCMAX=40-MU
      I1=M0+ISCMAX
      C INITIALIZATION COMPLETE
1000 CONTINUE
      IF (SC.EQ.0) GO TO 2400
      JSCFR=JSCFR+1
      IF (JSCFR.GT.JSCFR) GO TO 2400
      ML=N-L
      IF (ML.LE.1) GO TO 2400
      JSCFR=0
1050 DO 1060 J=1,N

```

12-18-71 15.614 RAM DECISION MODEL

SUBROUTINE RAM

```

1060 MS(J)=0
      NSIMP=NSIMP+1
      IF (L.EQ.0) GO TO 1076
      DO 1075 I=1,L
      J=IAHS(S(I))
1075 MS(J)=-S(I)
      IF (NOP.NE.2) GO TO 1076
      PRINT 3600,((S(K),SB(K)),K=1,L)
C     CALL THE IMBEDDED LINEAR PROGRAM
1076 CALL SIMPLE(II,N,M0,A,C,B,KO,XL,D,JH,XX,Y,OBJ,NOP)
      IF (NSIMP.NE.1) GO TO 1090
      PRINT 9999
      PRINT 1078,OBJ
1078 FORMAT(/////14X,13HLP SOLUTION =,F13.4)
1090 II = II + IPOST
C     KO(1) EQ 0 MEANS OBJ LESS THAN ZBAR
C     EQ 2 MEANS INFINITY
C     EQ 4 MEANS TROUBLE
C     EQ 6 MEANS OBJ GF ZBAR
      IF (KO(1).EQ.2) GO TO 3700
      IF (KO(1).EQ.4) GO TO 500
      IF (KO(1).EQ.6) GO TO 1500
      VLP5=-OBJ
      IF (VLP5.LE. (-ZBAR)) GO TO 1499
1349 DO 1350 I=1,N
      IF (D(I).NE.AINT(D(I)).AND.NS(I).NE.0) GO TO 1500
1350 CONTINUE
      DO 1450 J=1,N
      IF (NS(J).EQ.0) GO TO 1450
      I = J
      L = L + 1
      NS(J) = 0
      SU(L) = RCIB
      IF (D(I).NE.0.0) GO TO 1400
      IXINS(J) = -J
      S(L) = -J
      GO TO 1450
1400 S(L) = J
      NSS = (J - 1)/KSS + 1
      JSS(NSS) = 1
      IXINS(J) = J
      ZS = ZS + C(J)
      DO 1425 II=1,M
1425 BS(II) = DS(II) + A(II,J)
1450 CONTINUE
      GO TO 2329
1499 RC(1)=0
C     COMPUTE NEW SURROGATE CONSTRAINT
1500 IF (ISGMAX.LI.0) GO TO 1599
      BHP1=ZBAR

```

12-18-71 16.614 RAM DECISION MODEL

SUBROUTINE RAM

```

      DO 1505 I=1,MU
1505  HMP1=HMP1+XL(I)*B(I)
      IF (ABS(HMP1-B(M)).LE.0.0005) GO TO 1599
      IF (M-MU.LT.ISCMAX) GO TO 1520
      DO 1510 I=M1,M
      B(I)=B(I+1)
      BS(I)=BS(I+1)
      DO 1510 J=1,N
1510  A(I,J)=A(I+1,J)
      M=M+1
1520  B(M+1)=HMP1
      DO 1550 J=1,N
      ZJH=XX(J)
      IF (JH(J).GE.(-N)) ZJH=-ZJH
      IF (JH(J).GT.0) ZJH=0.
1550  A(M+1,J)=ZJH
      M=M+1
      HS(M)=B(M)
      DO 1575 K=1,L
      K1=S(K)
      IF (K1.LE.0) GO TO 1575
      BS(M)=BS(M)+A(M,K1)
1575  CONTINUE
      IF (NOP.NE.3) GO TO 1599
      PRINT 1598,M
      PRINT 9600,(A(M,J),J=1,N),B(M),BS(M)
1598  FORMAT (22HUSURROGATE CONSTRAINTS,2X,14)
1599  IF(KO(1).EQ.6)GO TO 3700
C      CHECK THE ROUNDED DUAL SOLUTION FOR FEASIBILITY
1900  CONTINUE
      TD=.5
900   F=ZS
      F1=BS(M)
905   DO 910 J=1,N
      IF (NS(J).EQ.0) GO TO 910
      IF (D(J).LT.TD) GO TO 910
      F=F+C(J)
      F1=F1+A(M,J)
910   CONTINUE
      IF (F.GE.ZBAK) GO TO 2400
      IF (F1.GE.0.0) GO TO 920
915   GO TO 2400
920   DO 925 I=1,MU
      F2=BS(I)
      DO 925 J=1,N
      IF (NS(J).EQ.0)GO TO 925
      IF (D(J).LT.TD) GO TO 925
      F2=F2+A(I,J)
925   CONTINUE
      IF (F2.LT.0.0) GO TO 915

```

12-18-71 16.614 RAM DECISION MODEL

SUBROUTINE RAM

```

930  CONTINUE
C    ROUNDED DUAL SOLUTION FEASIBLE
      NOPT = NOPT + 1
      IF (M.EQ.MU) GO TO 940
      DO 935 I=1,M
        H(I) = B(I)*F-ZKBAR-ZBAR
935  HS(I) = BS(I)*F-ZKBAR-ZBAR
940  ZBAR = F-ZKBAR
      DO 945 J=1,L
        SHAXH(J) = SB(J)
945  SHAX(J) = S(J)
      K=L
      DO 950 J=1,N
        IF (NS(J).EQ.0) GO TO 950
        K=K+1
        SHAXH(K) = ULANK
        SHAX(K) = J
        IF (D(J).LT.10) SHAX(K) = -J
950  CONTINUE
      PRINT 9992
9302 PRINT 3310,NOPT,F
9304 K = 0
      DO 3303 I=1,N
        IF (SHAX(I).LE.0) GO TO 3303
        K = K + 1
        T(K) = SHAX(I)
9303 CONTINUE
      PRINT 3600,(T(J),J=1,K)
      NOBJ = OBJ
      ZOBJ = NOBJ
      IF (OBJ.NE.ZOBJ) ZOBJ = ZOBJ*1.0
      IF (F.EQ.ZOBJ) GO TO 3700
      GO TO 2400
C    BEGINNING OF AN ITERATION
9410 IF (ZS.GE.ZBAR) GO TO 3700
      I1 = ICON + 1
9450 IF (HS(I1).LT.0.0) GO TO 1980
      I1 = ICON
      IF (HS(I1).LT.0.0) GO TO 3700
      GO TO 2320
9480 CONTINUE
C    TEST 1 AND 2
      DO 2000 J=1,N
        IF (NS(J).EQ.0) GO TO 2000
        IF (ZS+C(J).GE.ZBAR) GO TO 1999
      DO 1998 I=1,MU
        IF (I.GT.ICON1.AND.I.LE.(ICON+1)) GO TO 1998
        IF (BS(I)*A(I,J).LT.0.0) GO TO 1999
9498 CONTINUE
      GO TO 2000

```

12-18-71 16.614 RAM DECISION MODEL

SUBROUTINE RAM

```

1999 NS(J) = 0
   L = L + 1
   SB(L) = HCU8
   S(L) = -J
   IXINS(J) = -J
2000 CONTINUE
C   TEST 3 AND 4
   K1 = 0
   ZTEST = Z5
   DO 1921 I=1,ISS
   IF(JSS(I).EQ.1)GO TO 1921
   K1 = K1 + 1
   T(K1) = 1
   IK = 1*KSS - (KSS - 1)
   JK = 1*KSS
   CMIN(1) = 1.0E10
   DO 1923 J2=IK,JK
   IF(NS(J2).EQ.0)GO TO 1923
   IF(C(J2).LT.CMIN(1))CMIN(1)=C(J2)
1923 CONTINUE
   ZTEST = ZTEST + CMIN(1)
1921 CONTINUE
   IF(ZTEST.GE.ZBAR)GO TO 3700
   DO 1924 I1=1,K1
   K2 = T(I1)
   ZTEST = ZTEST - CMIN(K2)
   IA = K2*KSS - (KSS - 1)
   JK = K2*KSS
   DO 1925 I2=IK,JK
   IF(NS(I2).EQ.0)GO TO 1925
   IF(ZTEST+C(I2).LT.ZBAR)GO TO 1925
   NS(I2) = 0
   L = L + 1
   SB(L) = HCU8
   S(L) = -I2
   IXINS(I2) = -I2
1925 CONTINUE
   ZTEST = ZTEST + CMIN(K2)
1924 CONTINUE
C   TEST 5 AND 6
   DO 1930 J=1,M
   U = BS(J)
   DO 1931 I=1,K1
   K2 = I(1)
   IA = K2*KSS - (KSS - 1)
   JK = K2*KSS
   AMAX(K2) = -1.0E10
   DO 1933 K3=IK,JK
   IF(NS(K3).EQ.0)GO TO 1933
   IF(A(J,K3).GT.AMAX(K2))AMAX(K2)=A(J,K3)

```

12-18-71 16.614 RAM DECISION MODEL

SUBROUTINE RAM

```

1933 CONTINUE
  Q = Q + AMAX(K2)
1931 CONTINUE
  IF (Q.LT.0.0) GO TO 3700
  DO 1934 I=1,K1
    K4 = I(I)
    Q = Q - AMAX(K4)
    JK = K4*KSS - (KSS - 1)
    JK = K4*KSS
    DO 1935 I2=1K,JK
      IF (NS(I2).EQ.0) GO TO 1935
      IF (Q+A(I2).GE.0.0) GO TO 1935
      NS(I2) = Q
      L = L + 1
      SH(L) = BCIB
      S(L) = -I2
      IX(NS(I2)) = -I2
1935 CONTINUE
  Q = Q + AMAX(K4)
1934 CONTINUE
1930 CONTINUE
C  TEST /
  DO 1940 I=1,K1
    L1 = 0
    K2 = I(I)
    JK = K2*KSS - (KSS - 1)
    JK = K2*KSS
    DO 1941 I2=1K,JK
      IF (NS(I2).EQ.0) GO TO 1941
      L1 = L1 + 1
      L2 = I2
1941 CONTINUE
    IF (L1.NE.1) GO TO 1940
    IF (ZS+C(L2).GE.ZBAK) GO TO 3700
    DO 1943 I2=1,M
      IF (I2.LE.ICON1) GO TO 1944
      IF (I2.GT.ICON.AND.I2.LE.(ICON+ISS+1)) GO TO 1943
      Q = HS(I2) + A(I2,I2)
      DO 1945 I4=1,ISS
        IF (JSS(I4).EQ.1) GO TO 1945
        IF (I4.EQ.K2) GO TO 1945
        AMAX(I4) = -1.0E10
        JK = I4*KSS - (KSS - 1)
        JK = I4*KSS
        DO 1946 I6=1K,JK
          IF (NS(I6).EQ.0) GO TO 1946
          IF (A(I2,I6).GT.AMAX(I4)) AMAX(I4) = A(I2,I6)
1946 CONTINUE
        Q = Q - AMAX(I4)
1945 CONTINUE

```

12-18-71 16.614 RAM DECISION MODEL

SUBROUTINE RAM

```

      IF (U.LT.0.0) GO TO 3700
      GO TO 1943
1944 IF (BS(12)+A(12,L2).LT.0.0) GO TO 3700
1943 CONTINUE
      NS(L2) = 0
      L = L + 1
      SB(L) = BCIM
      S(L) = L2
      IXINS(L2) = L2
      JSS(K2) = 1
      ZS = ZS + C(L2)
      DO 1942 J=1,M
1942 BS(J) = BS(J) + A(J,L2)
1940 CONTINUE
      IF (ZS.GE.ZBAR) GO TO 3700
      I1 = ICON + 1
      IF (BS(I1).LT.0.0) GO TO 1000
C      A BETTER SOLUTION HAS BEEN FOUND
2320 CONTINUE
      IF (M.EQ.M0) GO TO 2340
C      REVISE B(I) AND BS(I) USING NEW ZS
      DO 2325 I=M1,M
      B(I)=B(I)+ZS-ZKBAR-ZHAR
2325 BS(I)=BS(I)+ZS-ZKBAR-ZBAR
2340 ZHAR=ZS-ZKBAR
      DO 2350 J=1,N
2350 SHAX(J)=S(J)
      GO TO 3300
C      AUGMENTATION STEP
2400 K1=0
      IF (SC.EQ.0) GO TO 2415
      IF (IFIRST.NE.0) GO TO 2415
      IFIRST=1
      DO 2410 J=1,N
      IF (NS(J).EQ.0) GO TO 2410
      J1=0
      IF (D(J).GE.0.9999) J1=J
      IF (J1.EQ.0) GO TO 2410
      J2 = (J1 - 1)/KSS + 1
      IF (JSS(J2).EQ.1) GO TO 2410
      L=L+1
      NS(J)=0
      S(L)=J1
      IXINS(J) = J1
      ZS=ZS+C(J)
      JSS(J2) = 1
      DO 2405 I=1,M
2405 BS(I)=BS(I)+A(I,J)
2410 CONTINUE
2415 CONTINUE

```

12-18-71 16.614 RAM DECISION MODEL

SUBROUTINE RAM

```

      DO 2500 J=1,ISS
      IF (JSS(J).EQ.1) GO TO 2500
      K1=K1+1
      T(K1) = J
2500  CONTINUE
      IF (K1.NE.0) GO TO 2432
      IF (ZS.GE.ZBAR) GO TO 3700
      GO TO 2320
CHAR7*****AUG WITH MIN C FROM SS MAX(CMAX-CMIN)*****
2432  J1 = 0
      J2 = 0
      DMAX = 0.0
      DO 2430 K2=1,K1
      CMAX = 0.0
      CMIN = 1.0E10
      JS = T(K2)
      IK = JS*KSS - (KSS - 1)
      JK = JS*KSS
      DO 2429 K=IK,JK
      IF (NS(K).EQ.0) GO TO 2429
      IF (C(K).GT.CMAX) CMAX=C(K)
      IF (C(K).LT.CMIN) CMIN=C(K)
2429  CONTINUE
      DIFF = CMAX - CMIN
      IF (DIFF.LT.DMAX) GO TO 2430
      DMAX = DIFF
      J2 = J3
2430  CONTINUE
      IF (J2.EQ.0) GO TO 3700
      XMIN = 1.0E10
      IK = J2*KSS - (KSS - 1)
      JK = J2*KSS
      DO 2431 K=IK,JK
      IF (NS(K).EQ.0) GO TO 2431
      IF (C(K).GE.XMIN) GO TO 2431
      XMIN = C(K)
      J1 = K
2431  CONTINUE
      DO 2433 I=1,N
      IF (I.LE.ICON1) GO TO 2436
      IF (I.GT.ICON.AND.I.LE.(ICON+ISS+1)) GO TO 2433
      Q = BS(I) + A(I,J1)
      DO 6000 L1=1,K1
      JS = T(L1)
      IF (JS.EQ.J2) GO TO 6000
      IK = JS*KSS - (KSS-1)
      JK = JS*KSS
      AMAX(JS) = -1.0E10
      DO 6001 L2=IK,JK
      IF (NS(L2).EQ.0) GO TO 6001

```

12-16-71 16.614 RAM DECISION MODEL

SUBROUTINE RAM

```

      IF(A(I,L2).GT.AMAX(J5))AMAX(J5)=A(I,L2)
6001 CONTINUE
      Q = Q + AMAX(J5)
6000 CONTINUE
      IF(Q.GE.U.0)GO TO 2433
      GO TO 2435
2436 IF(RS(I)+A(I,J1).GE.U.0)GO TO 2433
      GO TO 2435
2433 CONTINUE
      IF(ZS+C(J1).LT.ZBAR)GO TO 2434
2435 NS(J1) = 0
      L = L + 1
      SH(L) = 8C18
      IXINS(J1) = -J1
      S(L) = -J1
      GO TO 2432
2434 NS(J1) = 0
      L = L + 1
      IXINS(J1) = J1
      S(L) = J1
      J2 = (J1 - 1)/KSS + 1
      JSS(J2) = 1
      ZS = ZS + C(J1)
      DO 2600 I=1,M
2600 RS(I) = RS(I) + A(I,J1)
      IF(NOP.EQ.5)GO TO 1910
      K1 = 0
      DO 2610 I=1,ISS
      IF(JSS(I).EQ.1)GO TO 2610
      K1 = K1 + 1
      T(K1) = I
2610 CONTINUE
      IF(K1.NE.0)GO TO 2432
      IF(ZS.GE.ZBAR)GO TO 3700
      GO TO 2320
3300 NOPT=NOPT+1
      PRINT9992
      PRINT3310,NOPT,ZS
3310 FORMAT(14X,13,1X,23HFEASIBLE SOLUTION FOUND,3X,2HZF,
      *F13.4)
3305 K = 0
      DO 3301 I=1,L
      IF(S(I).LE.0)GO TO 3301
      K = K + 1
      T(K) = S(I)
3301 CONTINUE
3304 PRINT 3600,(T(J),J=1,K)
3600 FORMAT(14X,1015)
3700 NFATH=NFATH+1
3710 IF(SB(L).EQ.BLANK)GO TO 3900

```

12-18-71 16.614 RAM DECISION MODEL

SUBROUTINE RAM

```

      J=IABS(S(L))
      NS(J)=J
      IXINS(J) = 0
      IF (S(L).LT.0) GO TO 3735
      J2 = (J - 1)/KSS + 1
      JSS(J2) = 0
      ZS=ZS-C(J)
      DO 3725 I=1,M
3725  BS(I)=BS(I)-A(I,J)
3735  S(L)=BLANK
      S(L)=0
      L=L-1
      IF (L.GT.0) GO TO 3710
C      FINISHED
      PRINT 9993
      PRINT 3750
3750  FORMAT(/////11X,30H IMPLICIT ENUMERATION COMPLETE)
      PRINT 3739,RISK(NHUN)
3739  FORMAT(//11X,25H COEFFICIENT OF RISK IS =,F9.6)
      NITER=NHATH+NIATH-1
      PRINT 3850,NOPT,NSIMP,NITER
3850  FORMAT(12X,22HNO. FEASIBLE SOLUTIONS,15/
1      11X,10H LP CALLED,15,6H TIMES/
2      11X,15H NO. ITERATIONS,18)
      DO 3740 J=1,N
3740  S(J)=0
      DO 3742 J=1,N
      K=IABS(SMAX(J))
      IF (K.EQ.0) GO TO 3744
3742  S(K)=1
3744  DO 3746 K=1,N
      IF (S(K).NE.0) GO TO 3746
      SMAX(J)=-K
      J=J+1
3746  CONTINUE
      ZBAR=0.0
      DO 3835 J=1,N
      K=IABS(SMAX(J))
      IF (CS(K).LT.0.0) SMAX(J)=-SMAX(J)
      IF (SMAX(J).GT.0) ZBAR=ZBAR+CS(K)
3835  CONTINUE
      PRINT 3840,ZBAR
3840  FORMAT(11X,28H THE OPTIMAL VALUE OF Z IS =,F13.4)
3800  DO 3810 K=1,N
3810  T(K)=0
      DO 3820 K=1,N
      K1=IABS(SMAX(K))
3820  IF (SMAX(K).GT.0) T(K1)=K1
      PRINT 3830,(T(K),K=1,N)
3830  FORMAT(9(1X,13))

```

12-18-71 16.614 RAM DECISION MODEL

SUBROUTINE RAM

```

      IF(NOP.NE.1)GO TO 500
      DO 3833 I=1,ICON
      TT = 0.0
      DO 3831 J=1,N
      IF(T(J).GT.0) TT=TT+A(I,J)
3831 CONTINUE
      PRINT 3832,TT
3832 FORMAT(F10.4)
3833 CONTINUE
C      END OF OUTPUT. LOOK FOR ANOTHER PROBLEM NOW
500 RETURN
C      COMPLEMENT AND UNDERScore LAST REMAINING ENTRY IN S.
3900 SB(L)=BCIB
      S(L)=-S(L)
      J=IABS(S(L))
      IF (S(L).GT.0) GO TO 3950
      J2 = (J - 1)/KSS + 1
      J3 = J2*KSS - (KSS - 1)
      J4 = J2*KSS
      DO 3901 K=J3,J4
      IF(K.EQ.J)GO TO 3901
      IF(IXINS(K).EQ.(-K))GO TO 3901
      JSS(J2) = 0
      ZS=ZS-C(J)
      IXINS(J) = -J
      DO 3925 I=1,M
3925 RS(I)=RS(I)-A(I,J)
      GO TO 1910
3901 CONTINUE
      S(L) = J
      GO TO 3710
3950 PRINT 3951
3951 FORMAT(21H ERROR NONE BCIB ZERO)
      GO TO 500
      END

```

6 WORDS OF MEMORY USED BY THIS COMPILATION

12-18-71 16.642 RAM DECISION

SUBROUTINE SIMPLE

```

C          RAM DECISION
C          SUBROUTINE SIMPLE
C          REDUNDANT EQUATIONS CAUSE INFEASIBILITY
C          SUBROUTINE SIMPLE(INFLAG,MX,NN,A,B,C,KO,XL,P,JH,X,Y,
C          •OHJ,NUP)
C          REAL B(1),C(1),XL(1),P(1),X(1),Y(1)
C          INTEGER INFLAG,MX,NN,KO(6),KO(180),JH(1)
C          REAL L(180,180)
C          REAL A(40,180)
C          DIMENSION NI(180)
C          REAL AA,A1JT,BU,COST,UT,RCOST,TEXP,TPIV,TY,XOLD,XX,X
C          •Y,YI,YMAX,EM
C          INTEGER I,IA,INVC,IR,ITER,J,JT,K,KBJ,LL,M,N,JT2
C          INTEGER NCUT,NUMVR,NVER,NUMPV
C          LOGICAL TRIG,VER
C          LOGICAL FINV,FFRZ,SCH
C          COMMON MS(180),ZBAR,LPSEQ
C          SET INITIAL VALUES, SET CONSTANT VALUES
C          FINV=.FALSE.
C          TRIG=.FALSE.
C          ITER=0
C          LPSEQ=LPSEQ+1
C          NUMVR=0
C          NUNPV=0
C          M=MX
C          N=NN
C          TEXP=.5*.16
C          NVER=M/2+5
C          NCUT=4*M+10
C          IF(INFLAG.EQ.0) GO TO 1410
C          IMPOSE CORRECT TEMPERATURE ON ROWS
C          FFRZ=.TRUE.
C          DO 1960 L=1,M
C          100 IF(MS(L).EQ.NF(L))GO TO 1955
C          IF(MS(L)*NF(L).GT.0.0H.(MS(L).EQ.0.AND.X(L).GE.0.))
C          •GO TO 1950
C          I=L
C          IF(NF(L).NE.0) GO TO 1925
C          1920 IF(JH(I).GT.0) GO TO 1930
C          IF(MS(L).GT.0.AND.JH(L).GE.(-M)) GO TO 1950
C          IF(MS(L).LT.0.AND.JH(L).LT.(-M)) GO TO 1950
C          1925 DO 1926 J=1,M
C          P(J)=P(J)+E(I,J)
C          E(I,J)=-E(I,J)
C          1926 CONTINUE
C          OJJ=OHJ+X(I)
C          X(I)=-X(I)
C          JHL=JH(L)
C          IF(JHL.LE.(-M)) JH(L)=-L-M
C          IF(JHL.LT.(-M)) JH(L)=-L

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12-18-71 16.642 RAM DECISION

SUBROUTINE SIMPLE

```

      GO TO 1950
1930  JI=-1
      COST=P(1)
      IF (MS(1).GT.0) GO TO 1931
      JI=JT-M
      COST=1.-COST
1931  EN=1.
      GO TO 630
C     GET COLUMN(JT)
1932  SCH=.FALSE.
      IF (COST.GT.0.) GO TO 1938
1935  GO TO 1000
C     SELECT ROW(IR)
1936  IF (IR.NE.0.OR.SCH) GO TO 1940
      SCH=.TRUE.
1938  EN=-EN
      DO 1937 J=1,M
      Y(J)=-Y(J)
1937  CONTINUE
      GO TO 1935
1940  IF ((SCH.AND.ABS(COST).GT.IPIV).OR.IR.EQ.0) GO TO 198
      *0
1941  IF (EN.GT.0.) GO TO 1945
      DO 1942 J=1,M
      Y(J)=-Y(J)
1942  CONTINUE
1945  GO TO 901
C     PIVOT(IR,JT)
1950  NF(1)=MS(1)
1955  IF (JH(1).LT.0) GO TO 1960
      IA=JH(1)
      KB(IA)=1
1960  CONTINUE
      FFRZ=.FALSE.
      GO TO 910
C*   START WITH SINGLETON BASIS
1410  DO 1402 J=1,N
      KB(J)=0
1402  CONTINUE
      FFRZ=.FALSE.
1400  DO 1401 I=1,M
      JH(1)=-1
      NF(1)=MS(1)
      IF (NF(1).LT.0.OR.(NF(1).EQ.0.AND.B(1).LT.0.)) JH(1)=
      * -1-M
1401  CONTINUE
C*   CREATE INVERSE FROM KB AND JH      (STEP 7)
1320  VER=.TRUE.
      INVC=0
      NUMVR=NUMVR+1

```

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SUBROUTINE SIMPLE

```

      TRIG=.FALSE.
      OBJ=0.
      DO 1113 I=1,M
      DO 1151 J=1,M
      E(J,I)=0.
1151 CONTINUE
      IF (JH(I).LT.(-M)) GO TO 1111
      IF (JH(I).GT.0) JH(I)=0
      E(I,I)=1.
      P(I)=0.
      X(I)=B(I)
      GO TO 1113
1111 E(I,I)=-1.
      P(I)=+1.
      OBJ=OBJ+H(I)
      X(I)=-B(I)
1113 CONTINUE
      DO 1102 JI=1,N
      IF (KB(JI).EQ.0) GO TO 1102
      GO TO 600
1114 TY=TPIV
      IR=0
      COSI=C(JT)
      DO 1104 I=1,M
      COSI=COSI+A(JT,I)*P(I)
      IF (JH(I).NE.0.OR.X(I).NE.0..OR.ABS(Y(I)).LE.TY) GO TO 1104
      TY=ABS(Y(I))
      IR=I
1104 CONTINUE
      IF (IR.NE.0) GO TO 1119
      TY=0.
      DO 1105 I=1,M
      IF (JH(I).NE.0.OR.X(I).EQ.0..OR.ABS(Y(I)).LE.TPIV) GO TO 1105
      IF (ABS(Y(I)).LE.TY*ABS(X(I))) GO TO 1105
      TY=ABS(Y(I)/X(I))
      IR=I
1105 CONTINUE
1119 IF (IR.NE.0) GO TO 900
C   PIVOT(IR,JT)
      FINV=.TRUE.
      IF (NOP.EQ.0) PRINT 1199,LPSEQ
1199 FORMAT(15HINVERT FAIL LP,14)
      GO TO 1410
1102 CONTINUE
C   PERFORM A SIMPLEX ITERATION
1200 VER=.FALSE.
      DO 500 I=1,M
      IF (NF(I).EQ.0.AND.X(I).LT.0.) X(I)=0.

```

12-18-71 16.642 RAM DECISION

SUBROUTINE SIMPLC

```

503 CONTINUE
C* FIND MINIMUM REDUCED COST (STEP 3)
599 JT=0
   BU=0.0
   DO 701 J=1,N
   IF (KB(J).NE.0) GO TO 701
   DT=C(J)
   DO 303 I=1,M
   DT=DT+A(J,I)*P(I)
303 CONTINUE
   IF (DT.GE.BU) GO TO 701
   BU=DT
   JT=J
701 CONTINUE
   DO 702 I=1,M
   IF (JH(I).LT.0) GO TO 702
   IF (P(I).LT.BU) GO TO 703
   IF ((1.-P(I)).GE.BU) GO TO 702
   BU=1.-P(I)
   JI=-I-M
   GO TO 702
703 BU=P(I)
   JI=-I
702 CONTINUE
   COST = BU
   IF (JT.EQ.0) GO TO 203
   IF (ITER.GE.NCUT) GO TO 160
   ITER = ITER + 1
   IF (JT.LT.0) GO TO 630
C* BEGIN SUBROUTINE GET COLUMN(JT)
600 DO 610 I=1,M
   Y(I)=0.0
610 CONTINUE
   DO 605 I=1,M
   A1JT=A(JT,I)
   IF (A1JT.EQ.0.) GO TO 605
   DO 606 J=1,M
   Y(J)=Y(J)+A1JT*E(J,I)
606 CONTINUE
605 CONTINUE
   GO TO 640
630 JI2=-JT
   EM=1.
   IF (JI2.LE.M) GO TO 631
   JI2=JI2-M
   EM=-1.
631 DO 632 I=1,M
   Y(I)=EM*E(I,JI2)
632 CONTINUE
640 YMAX=0.

```

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SUBROUTINE SIMPLE

```

      DO 620 I=1,M
      YMAX=AMAX1(AHS(Y(I)),YMAX)
620  CONTINUE
      TPIV=YMAX*TEXP
C     END OF GET COLUMN
      IF (FFRZ) GO TO 1932
      IF (VER) GO TO 1114
      RCOST=YMAX/BB
      IF (TRIG.AND.HD.GE.(-TPIV)) GO TO 203
      TRIG=HB.GE.(-TPIV)
C*   SELECT PIVOT ROW                                (STEP 5)
1000  AA=TPIV
      IR=0
1002  DO 1003 I=1,M
      IF (X(I).NE.0..OR.Y(I).LE.AA.OR.NF(I).NE.0) GO TO 10
      • 03
      AA=Y(I)
      IR=I
1003  CONTINUE
      IF (IR.NE.0) GO TO 1020
      AA=0.
      DO 1010 I=1,M
      IF (NF(I).NE.0.OR.Y(I).LE.TPIV.OR.Y(I).LE.AA+X(I)) G
      • 0 TO 1010
      AA=Y(I)/X(I)
      IR=I
1010  CONTINUE
1020  IF (FFRZ) GO TO 1936
      IF (IR.EQ.0) GO TO 207
C*   PIVOT ON (IR,JT)                                (STEP 6)
901  IA=JH(IR)
      IF (IA.GT.0) KB(IA)=0
C   BEGIN SUBROUTINE PIVOT(IR,JT)
900  NOMPV=NOMPV+1
      JH(IR)=JT
      IF (JT.GT.0) KB(JT)=IR
      YI=-Y(IR)
      Y(IR)=-1.0
      DO 904 J=1,M
905  XY = - (IR,J)/YI
      IF (XY.EQ.0.) GO TO 904
      P(J)=P(J)+COST*XY
      E(IR,J)=0.
      DO 906 I=1,M
911  E(I,J) = E(I,J) + XY*Y(I)
906  CONTINUE
904  CONTINUE
      XY=X(IR)/YI
      DO 908 I=1,M
      XOLD=X(I)

```

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SUBROUTINE SIMPLE

```

912 X(1) = XOLD + XY*Y(1)
908 CONTINUE
    Y(1R)=-Y1
    X(1R)=-XY
C    END OF PIVOT
    OBJ=OBJ+XY*COST
    IF (VER) GO TO 1102
C    EXCHANGE ROWS IF SLACK PIVOTED IN WRONG ROW
    IF (J1.GT.0.OR.JT2.EQ.1R) GO TO 907
    XY=X(1R)
    X(1R)=X(J12)
    X(J12)=XY
    DO 909 I=1,M
    XY=E(1R,I)
    E(1R,I)=E(J12,I)
    E(J12,I)=XY
909 CONTINUE
    IA=JH(J12)
    JH(J12)=J1
    JH(1R)=IA
    KB(IA)=1R
907 INVC=INVC+1
C    TO STEP 1 IF NOT INVERTING, TO STEP 7 IF INVERTING
    IF (FFRZ) GO TO 1950
    IF (QJHJ.GE.ZBAR) GO TO 180
    IF (FINV) GO TO 1200
910 IF (INVC.GE.NVER) GO TO 1320
    GO TO 1200
C*    END OF ALGORITHM, SET EXIT VALUES    ...
207 IF (RCOST.LE.(-1000.)) GO TO 203
C    INFINITE SOLUTION
    K=2
    GO TO 250
180 K=6
    GO TO 250
C    PROBLEM IS CYCLING PERHAPS
160 K=4
    PRINT 161,LPSEQ
161 FORMAT (31H011TERATION LIMIT EXCEEDED ON LP,14)
    GO TO 250
C    FEASIBLE OR INFEASIBLE SOLUTION
203 K=0
250 DO 1399 J=1,N
    XX=0.0
    KUJ=KB(J)
    IF (KBJ.NE.0) XX=X(KBJ)
    XL(J) = XX
1399 CONTINUE
    KU(1)=K
    KU(2)=11ER

```

12-18-71 16,642 RAM DECISION

SUBROUTINE SIMPLE

```

KU(3)=INVC
KU(4)=NUMVR
KU(5)=NUMPV
KU(6)=J1
RETURN
1980 IF (NOP.EQ.0) PRINT 1981,LPSEU,L,IR,SCH,COST
1981 FORMAT( 3HULP,14,12H FAIL, SLACK,13,4H IR=13,5H SCH=
•L1,5H C=,f19.6)
IF (IR.NE.0) GO TO 1941
GO TO 1410
END

```

7 WORDS OF MEMORY USED BY THIS COMPILATION

APPENDIX B
COMPUTER PROGRAM
FINAL OUTPUT

LP SOLUTION = 14413.9740

1 FEASIBLE SOLUTION FOUND Z= 14446.7753
 6 14 33 44 50 59 70 77 88 93
 105 113 123 133 151 166 21 156 142 177

2 FEASIBLE SOLUTION FOUND Z= 14435.7751
 6 14 33 44 50 59 70 77 88 93
 105 113 123 133 151 166 21 142 174 159

3 FEASIBLE SOLUTION FOUND Z= 14433.2877
 6 14 33 44 50 59 70 77 88 93
 105 113 123 133 151 166 142 174 156 23

4 FEASIBLE SOLUTION FOUND Z= 14430.5254
 6 14 33 44 50 59 70 77 88 93
 166 174 156 21 104 111 123 133 142 153

5 FEASIBLE SOLUTION FOUND Z= 14425.9380
 6 14 33 44 50 59 70 77 90 174
 156 166 140 23 92 105 133 113 123 153

6 FEASIBLE SOLUTION FOUND Z= 14423.1882
 6 14 33 44 50 59 70 77 90 174
 166 21 93 105 133 142 158 113 123 153

7 FEASIBLE SOLUTION FOUND Z= 14422.7883
 6 14 33 44 50 59 71 174 93 166
 140 23 77 105 133 156 90 113 151 122

8 FEASIBLE SOLUTION FOUND Z= 14422.1882
 6 14 33 44 50 60 174 93 133 166
 23 105 140 156 71 77 90 113 123 153

THE OPTIMAL VALUE OF Z IS = 14422.1877

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RESULTS FOR RISK LEVEL 1

COEFFICIENT OF RISK = 0.0004700
 MAXIMUM FAILURE RATE = 8500.0
 MAXIMUM WEIGHT = 6000.0
 MAXIMUM MEAN TIME TO REPAIR = 189.0

THE OPTIMAL SYSTEM RESULTS ARE

EXPECTED LIFE CYCLE COSTS = 13467.5
 VARIANCE OF LIFE CYCLE COSTS = 2031250.0
 STANDARD DEVIATION OF LIFE CYCLE COSTS = 1425.2

EXPECTED R AND D COSTS = 3462.5
 VARIANCE OF R AND D COSTS = 48920.0
 STANDARD DEVIATION OF R AND D COSTS = 221.2

EXPECTED FAILURE RATE = 7675.0
 VARIANCE OF FAILURE RATE = 111950.0
 STANDARD DEVIATION OF FAILURE RATE = 334.6

EXPECTED WEIGHT = 4923.0
 VARIANCE OF WEIGHT = 69899.0
 STANDARD DEVIATION OF WEIGHT = 264.4

EXPECTED MEAN TIME TO REPAIR = 156.1
 VARIANCE OF MEAN TIME TO REPAIR = 152.7
 STANDARD DEVIATION OF MTR = 12.4

EXPECTED AVAILABILITY = 0.9511
 VARIANCE OF AVAILABILITY = 0.00000937
 STANDARD DEVIATION OF AVAILABILITY = 0.0031

C

THE OPTIMAL SUBSYSTEM ALLOCATIONS ARE

SUBSYSTEM 1	LEVEL IS = 6
SUBSYSTEM 2	LEVEL IS = 5
SUBSYSTEM 3	LEVEL IS = 5
SUBSYSTEM 4	LEVEL IS = 0
SUBSYSTEM 5	LEVEL IS = 8
SUBSYSTEM 6	LEVEL IS = 5
SUBSYSTEM 7	LEVEL IS = 6
SUBSYSTEM 8	LEVEL IS = 8
SUBSYSTEM 9	LEVEL IS = 5
SUBSYSTEM 10	LEVEL IS = 9
SUBSYSTEM 11	LEVEL IS = 3
SUBSYSTEM 12	LEVEL IS = 6
SUBSYSTEM 13	LEVEL IS = 5
SUBSYSTEM 14	LEVEL IS = 6
SUBSYSTEM 15	LEVEL IS = 7
SUBSYSTEM 16	LEVEL IS = 5
SUBSYSTEM 17	LEVEL IS = 9
SUBSYSTEM 18	LEVEL IS = 9
SUBSYSTEM 19	LEVEL IS = 4
SUBSYSTEM 20	LEVEL IS = 3

SUBSYSTEM 1

LOC	EXPECTED VALUE ($\$ \times 10^{-6}$)	VARIANCE ($\$ 2 \times 10^{-12}$)	M AND D		VARIANCE ($\$ 2 \times 10^{-12}$)
			EXPECTED VALUE ($\$ \times 10^{-6}$)	EXPECTED VALUE ($\$ \times 10^{-6}$)	
	1100.0	40000.0	30.0		160.0
	1115.0	60000.0	53.0		250.0
	1000.0	80000.0	78.0		420.0
	1100.0	100000.0	75.0		500.0
	1050.0	150000.0	90.0		750.0
	1010.0	261000.0	105.0		950.0
	1060.0	380000.0	130.0		750.0
	1050.0	600000.0	140.0		900.0
	1000.0	810000.0	160.0		1300.0
M	EXPECTED VALUE (LGS)	VARIANCE (LHS2)	FM		VARIANCE (FAIL2/MN2)
			EXPECTED VALUE (FAILURES/MONTH)	EXPECTED VALUE (FAIL2/MN2)	
	50.0	300.0	100.0		200.0
	80.0	500.0	110.0		250.0
	100.0	675.0	130.0		350.0
	100.0	450.0	75.0		250.0
	125.0	700.0	85.0		300.0
	145.0	1200.0	105.0		400.0
	210.0	3600.0	50.0		350.0
	250.0	4000.0	60.0		400.0
	250.0	5000.0	80.0		500.0
	MTTR (MINUTES)	EXPECTED VALUE (MIN2)	VARIANCE		
			EXPECTED VALUE (MIN2)	EXPECTED VALUE (MIN2)	
	75.0		30.0		
	45.0		50.0		
	30.0		100.0		
	150.0		150.0		
	205.0		250.0		
	75.0		500.0		
	300.0		1000.0		
	240.0		1800.0		
	210.0		3000.0		

SUBSYSTEM 2

LOC	EXPECTED VALUE (SX10-6)	VARIANCE (SX10-12)	R AND U		VARIANCE (SX10-12)
			EXPECTED VALUE (SX10-6)	EXPECTED VALUE (SX10-12)	
	1100.0	40000.0	35.0	350.0	
	1075.0	70000.0	60.0	500.0	
	1060.0	80000.0	83.0	700.0	
	1040.0	130000.0	90.0	700.0	
	1030.0	149000.0	110.0	800.0	
	1020.0	275000.0	150.0	1000.0	
	1020.0	430000.0	115.0	1000.0	
	1000.0	600000.0	125.0	1400.0	
	900.0	622500.0	150.0	2000.0	
W	EXPECTED VALUE (LBS)	VARIANCE (LBS2)	FR		VARIANCE (FAIL2/MN2)
			EXPECTED VALUE (FAILURES/MONTH)	EXPECTED VALUE (FAIL2/MN2)	
	200.0	1000.0	400.0	500.0	
	250.0	1500.0	420.0	1000.0	
	350.0	2500.0	500.0	2000.0	
	290.0	1500.0	200.0	700.0	
	340.0	2600.0	220.0	1200.0	
	440.0	6400.0	380.0	2200.0	
	420.0	4900.0	100.0	1000.0	
	470.0	8100.0	120.0	2000.0	
	570.0	14400.0	200.0	4000.0	
MTIR	EXPECTED VALUE (MINUTES)	VARIANCE (MIN2)			
			EXPECTED VALUE (MINUTES)	EXPECTED VALUE (MIN2)	
	120.0	60.0	60.0	60.0	
	80.0	160.0	160.0	160.0	
	60.0	200.0	200.0	200.0	
	150.0	240.0	240.0	240.0	
	100.0	300.0	300.0	300.0	
	75.0	450.0	450.0	450.0	
	210.0	300.0	300.0	300.0	
	140.0	400.0	400.0	400.0	
	105.0	500.0	500.0	500.0	

SUBSYSTEM 3

LCC	EXPECTED VALUE (\$X10-6)	VARIANCE (\$X10-12)	R AND D	EXPECTED VALUE (\$X10-6)	VARIANCE (\$X10-12)
	1600.0	78000.0		58.0	580.0
	1695.0	100000.0		62.0	720.0
	1650.0	110000.0		113.0	960.0
	1720.0	180000.0		135.0	1400.0
	1650.0	230000.0		150.0	1800.0
	1640.0	322000.0		180.0	2180.0
	1630.0	830000.0		235.0	2580.0
	1600.0	1200000.0		245.0	3200.0
	1570.0	1400000.0		275.0	4300.0
W	EXPECTED VALUE (LBS)	VARIANCE (LBS2)	FN	EXPECTED VALUE (FAILURES/MONTH)	VARIANCE (FAIL2/MN2)
	400.0	4500.0		600.0	700.0
	450.0	6400.0		650.0	900.0
	475.0	8100.0		725.0	1300.0
	525.0	5600.0		300.0	1250.0
	575.0	8400.0		350.0	1600.0
	600.0	10000.0		450.0	2200.0
	740.0	12100.0		150.0	2500.0
	790.0	14400.0		200.0	4500.0
	815.0	16900.0		300.0	10000.0
MILK	EXPECTED VALUE (MINUTES)	VARIANCE (MIN2)			
	240.0	800.0			
	180.0	1000.0			
	150.0	1600.0			
	300.0	1600.0			
	225.0	2200.0			
	190.0	3200.0			
	450.0	3000.0			
	340.0	3600.0			
	280.0	4900.0			

SUBSYSTEM 4

LCC	EXPECTED VALUE (\$X10-6)	VARIANCE (\$2X10-12)	R AND D		VARIANCE (\$2X10-12)
			EXPECTED VALUE (\$X10-6)	EXPECTED VALUE (\$2X10-12)	
	1200.0	80000.0	35.0	300.0	
	1135.0	100000.0	60.0	500.0	
	1120.0	110000.0	83.0	650.0	
	1170.0	210000.0	75.0	600.0	
	1100.0	236000.0	82.5	800.0	
	1075.0	285000.0	100.0	950.0	
	1100.0	280000.0	135.0	1200.0	
	1075.0	750000.0	140.0	1500.0	
	1050.0	900000.0	160.0	2000.0	
W	EXPECTED VALUE (LMS)	VARIANCE (LMS2)	FR		VARIANCE (FAIL2/MN2)
			EXPECTED VALUE (FAILURES/MONTH)	EXPECTED VALUE (FAIL2/MN2)	
	80.0	750.0	220.0	150.0	
	90.0	850.0	250.0	200.0	
	120.0	1200.0	300.0	350.0	
	120.0	1100.0	100.0	300.0	
	130.0	1300.0	130.0	375.0	
	160.0	1500.0	200.0	500.0	
	180.0	1400.0	60.0	400.0	
	190.0	1650.0	90.0	800.0	
	220.0	2000.0	150.0	2000.0	
MYR	EXPECTED VALUE (MINUTES)	VARIANCE (MIN2)			
			EXPECTED VALUE (MINUTES)	EXPECTED VALUE (MIN2)	
	60.0	50.0			
	40.0	85.0			
	30.0	100.0			
	100.0	150.0			
	70.0	200.0			
	60.0	300.0			
	200.0	265.0			
	130.0	450.0			
	100.0	530.0			

SUBSYSTEM 5

LOC	EXPECTED VALUE (X10-6)	VARIANCE (X210-12)	R AND D		VARIANCE (X210-12)
			EXPECTED VALUE (X10-6)	EXPECTED VALUE (X210-12)	
	900.0	110000.0	50.0	170.0	
	850.0	295000.0	82.5	340.0	
	810.0	555000.0	125.0	1000.0	
	900.0	110000.0	50.0	170.0	
	850.0	295000.0	82.5	340.0	
	810.0	555000.0	125.0	1000.0	
	900.0	110000.0	50.0	170.0	
	850.0	295000.0	82.5	340.0	
	810.0	555000.0	125.0	1000.0	
	900.0	110000.0	50.0	170.0	
	850.0	295000.0	82.5	340.0	
	810.0	555000.0	125.0	1000.0	
M	EXPECTED VALUE (LRS)	VARIANCE (LHS2)	FR		VARIANCE (FAIL2/MN2)
			EXPECTED VALUE (FAILURES/MONTH)	EXPECTED VALUE (FAIL2/MN2)	
	100.0	650.0	300.0	7000.0	
	150.0	1000.0	350.0	12000.0	
	210.0	1300.0	500.0	22000.0	
	100.0	650.0	300.0	7000.0	
	150.0	1000.0	350.0	12000.0	
	210.0	1300.0	500.0	22000.0	
	100.0	650.0	300.0	7000.0	
	150.0	1000.0	350.0	12000.0	
	210.0	1300.0	500.0	22000.0	
NTR	EXPECTED VALUE (MINUTES)	VARIANCE (MIN2)	NTR		VARIANCE (MIN2)
			EXPECTED VALUE (MINUTES)	EXPECTED VALUE (MIN2)	
	360.0	1500.0	360.0	1500.0	
	300.0	2000.0	300.0	2000.0	
	240.0	3000.0	240.0	3000.0	
	360.0	1500.0	360.0	1500.0	
	300.0	2000.0	300.0	2000.0	
	240.0	3000.0	240.0	3000.0	
	360.0	1500.0	360.0	1500.0	
	300.0	2000.0	300.0	2000.0	
	240.0	3000.0	240.0	3000.0	

SUBSYSTEM 6

LCC	EXPECTED VALUE (SXI0-6)	VARIANCE (SXI0-12)	R	AMU	U	EXPECTED VALUE (SXI0-6)	VARIANCE (SXI0-12)
	1140.0	30000.0				90.0	350.0
	1075.0	60000.0				75.0	750.0
	1050.0	75000.0				100.0	1250.0
	1050.0	160000.0				120.0	1050.0
	900.0	190000.0				150.0	1500.0
	950.0	290000.0				200.0	2200.0
	980.0	400000.0				260.0	2500.0
	900.0	660000.0				225.0	3000.0
	915.0	940000.0				250.0	4000.0
M	EXPECTED VALUE (LHS)	VARIANCE (LHS2)	FK	EXPECTED VALUE (FAILURES/MONTH)	VARIANCE (FAIL2/MN2)		
	150.0	275.0		500.0	1000.0		
	220.0	500.0		600.0	2000.0		
	340.0	800.0		750.0	5000.0		
	200.0	500.0		200.0	1200.0		
	270.0	725.0		300.0	3000.0		
	390.0	1025.0		450.0	7000.0		
	300.0	2500.0		125.0	1000.0		
	370.0	6400.0		225.0	4000.0		
	490.0	10000.0		300.0	9000.0		
MTIR	EXPECTED VALUE (MINUTES)	VARIANCE (MIN2)					
	50.0	25.0					
	40.0	50.0					
	30.0	100.0					
	80.0	50.0					
	60.0	100.0					
	50.0	200.0					
	120.0	100.0					
	95.0	200.0					
	80.0	400.0					

SUBSYSTEM 7

LOC	EXPECTED VALUE		VARIANCE		K AND U		VARIANCE	
	(3X10-6)	(32X10-12)	(3X10-6)	(32X10-12)	(3X10-6)	(32X10-12)	(3X10-6)	(32X10-12)
	1700.0	40000.0			30.0	160.0		
	1610.0	65000.0			67.5	320.0		
	1600.0	80000.0			100.0	600.0		
	1540.0	200000.0			67.5	1100.0		
	1520.0	270000.0			105.0	1200.0		
	1500.0	350000.0			125.0	1300.0		
	1540.0	810000.0			135.0	1100.0		
	1505.0	1340000.0			142.5	1400.0		
	1465.0	1500000.0			185.0	2200.0		
W	EXPECTED VALUE		VARIANCE		FK		VARIANCE	
	(LHS)	(LHS2)	(LHS2)	(LHS2)	(FAILURES/MONTH)	(FAIL2/MN2)	(FAIL2/MN2)	(FAIL2/MN2)
	50.0	250.0			160.0	100.0		
	60.0	350.0			190.0	200.0		
	70.0	500.0			250.0	500.0		
	60.0	400.0			80.0	250.0		
	70.0	500.0			200.0	400.0		
	83.0	700.0			180.0	750.0		
	80.0	650.0			45.0	300.0		
	100.0	725.0			75.0	500.0		
	130.0	900.0			100.0	1000.0		
MTTR	EXPECTED VALUE		VARIANCE					
	(MINUTES)	(MIN2)	(MIN2)	(MIN2)				
	240.0	300.0						
	150.0	500.0						
	120.0	900.0						
	360.0	600.0						
	225.0	1000.0						
	180.0	1600.0						
	480.0	3200.0						
	300.0	2000.0						
	240.0	3600.0						

SUBSYSTEM N

LCC	EXPECTED VALUE (3X10-6)	VARIANCE (3X10-12)	K AND D		
			EXPECTED VALUE (3X10-6)	EXPECTED VALUE (3X10-12)	VARIANCE (3X10-12)
	960.0	20000.0	80.0	80.0	850.0
	940.0	60000.0	100.0	100.0	930.0
	900.0	90000.0	135.0	135.0	1100.0
	910.0	200000.0	112.5	112.5	1200.0
	910.0	150000.0	125.0	125.0	1500.0
	900.0	200000.0	175.0	175.0	2400.0
	820.0	350000.0	166.0	166.0	2600.0
	810.0	510000.0	175.0	175.0	2200.0
	752500.0	752500.0	200.0	200.0	3000.0
W	EXPECTED VALUE (LBS)	VARIANCE (LBS2)	FR		
			EXPECTED VALUE (FAILURES/MONTH)	EXPECTED VALUE (FAIL2/MON2)	VARIANCE (FAIL2/MON2)
	160.0	2500.0	900.0	900.0	10000.0
	180.0	2800.0	1000.0	1000.0	12500.0
	210.0	3400.0	1200.0	1200.0	16000.0
	240.0	3800.0	500.0	500.0	11500.0
	260.0	3300.0	650.0	650.0	13500.0
	290.0	3900.0	850.0	850.0	18000.0
	300.0	4000.0	300.0	300.0	15000.0
	320.0	4300.0	360.0	360.0	17500.0
	360.0	4900.0	500.0	500.0	25000.0
MTIR	EXPECTED VALUE (MINUTLS)	VARIANCE (MIN2)			
			EXPECTED VALUE (MIN2)	EXPECTED VALUE (MIN2)	VARIANCE (MIN2)
	180.0	180.0	100.0	100.0	100.0
	90.0	90.0	150.0	150.0	150.0
	45.0	45.0	225.0	225.0	225.0
	240.0	240.0	140.0	140.0	140.0
	120.0	120.0	210.0	210.0	210.0
	60.0	60.0	380.0	380.0	380.0
	360.0	360.0	180.0	180.0	180.0
	180.0	180.0	270.0	270.0	270.0
	90.0	90.0	400.0	400.0	400.0

SUBSYSTEM 9

LCC		K AND U		VARIANCE	
EXPECTED VALUE (S2X10-6)	(S2X10-12)	EXPECTED VALUE (S2X10-6)	(S2X10-12)	EXPECTED VALUE (S2X10-6)	VARIANCE (S2X10-12)
1500.0	50000.0	37.5	130.0		
1495.0	70000.0	50.0	200.0		
1440.0	80000.0	100.0	500.0		
1400.0	100000.0	75.0	700.0		
1375.0	160000.0	180.0	900.0		
1350.0	275000.0	150.0	1800.0		
1400.0	880000.0	115.0	1500.0		
1350.0	1440000.0	120.0	2000.0		
1330.0	1600000.0	160.0	2700.0		
W		FR		VARIANCE	
EXPECTED VALUE (LMS)	VARIANCE (LRS2)	EXPECTED VALUE (FAILURES/MONTH)	(FAIL2/MN2)	EXPECTED VALUE (FAIL2/MN2)	VARIANCE
70.0	400.0	500.0	6000.0		
90.0	600.0	550.0	7000.0		
120.0	1100.0	650.0	9000.0		
100.0	800.0	350.0	7000.0		
120.0	1100.0	400.0	6000.0		
150.0	1600.0	500.0	10000.0		
160.0	1600.0	250.0	10000.0		
190.0	2200.0	300.0	21500.0		
250.0	3000.0	400.0	15000.0		
MTTR		VARIANCE		VARIANCE	
EXPECTED VALUE (MINUTES)	(MIN2)	EXPECTED VALUE (MIN2)	(MIN2)	EXPECTED VALUE (MINUTES)	VARIANCE
90.0	50.0	50.0			
60.0	75.0	75.0			
40.0	100.0	100.0			
120.0	100.0	100.0			
80.0	150.0	150.0			
60.0	200.0	200.0			
240.0	200.0	200.0			
160.0	300.0	300.0			
80.0	400.0	400.0			

SUBSYSTEM 10

LCC	EXPECTED VALUE (\$X10-6)	VARIANCE (\$2X10-12)	R AND D (\$X10-6)	EXPECTED VALUE (\$2X10-12)	VARIANCE (\$2X10-12)
	2200.0	80000.0	75.0	1380.0	
	2135.0	120000.0	180.0	1500.0	
	2120.0	150000.0	175.0	2000.0	
	2120.0	200000.0	150.0	2100.0	
	2075.0	250000.0	200.0	2600.0	
	2050.0	335000.0	300.0	5200.0	
	1840.0	900000.0	350.0	7000.0	
	1800.0	1320000.0	375.0	8000.0	
	1670.0	1600000.0	450.0	10000.0	
W	EXPECTED VALUE (LMS)	VARIANCE (LMS2)	FR	EXPECTED VALUE (FAILURES/MONTH)	VARIANCE (FAIL2/MN2)
	150.0	1000.0		660.0	700.0
	180.0	1250.0		720.0	1000.0
	220.0	1600.0		810.0	2000.0
	250.0	2000.0		420.0	2000.0
	290.0	2500.0		480.0	2500.0
	330.0	3500.0		600.0	4000.0
	400.0	6000.0		180.0	3000.0
	540.0	8500.0		200.0	5000.0
	500.0	10500.0		300.0	8000.0
MTTR	EXPECTED VALUE (MINUTES)	VARIANCE (MIN2)			
	72.0	30.0			
	50.0	70.0			
	36.0	150.0			
	100.0	90.0			
	75.0	160.0			
	60.0	225.0			
	180.0	150.0			
	130.0	200.0			
	100.0	290.0			

SUBSYSTEM 11

LCC	EXPECTED VALUE (\$X10-6)	VARIANCE (\$2X10-12)	M AND O	EXPECTED VALUE (\$X10-6)	VARIANCE (\$2X10-12)
	1470.0	60000.0		65.0	640.0
	1495.0	100000.0		100.0	1000.0
	1460.0	120000.0		150.0	2000.0
	1480.0	280000.0		125.0	1600.0
	1440.0	390000.0		150.0	2000.0
	1435.0	505000.0		250.0	4500.0
	1400.0	030000.0		230.0	3600.0
	1370.0	1300000.0		250.0	4500.0
	1355.0	1540000.0		310.0	5000.0
M	EXPECTED VALUE (LOS)	VARIANCE (LMS2)	FK	EXPECTED VALUE (FAILURES/MONTH)	VARIANCE (FAIL2/MN2)
	50.0	300.0		900.0	5000.0
	75.0	550.0		1050.0	9000.0
	100.0	1000.0		1250.0	15000.0
	130.0	1200.0		720.0	10000.0
	170.0	2000.0		610.0	15000.0
	220.0	3000.0		1980.0	25000.0
	300.0	4500.0		500.0	20000.0
	370.0	6500.0		600.0	25000.0
	500.0	10000.0		750.0	40000.0
	EXPECTED VALUE (MINUTES)	VARIANCE (MIN2)			
	120.0	200.0			
	100.0	500.0			
	90.0	900.0			
	240.0	750.0			
	200.0	1200.0			
	180.0	2000.0			
	450.0	1500.0			
	350.0	2500.0			
	320.0	5000.0			

SUBSYSTEM 12

LCC		R AND D	
EXPECTED VALUE (\$X10-6)	VARIANCE (\$X10-12)	EXPECTED VALUE (\$X10-6)	VARIANCE (\$X10-12)
1650.0	70000.0	60.0	350.0
1655.0	110000.0	75.0	700.0
1660.0	125000.0	125.0	650.0
1665.0	310000.0	120.0	2000.0
1750.0	360000.0	150.0	2500.0
1720.0	405000.0	250.0	4000.0
1700.0	780000.0	225.0	5000.0
1700.0	1300000.0	275.0	4500.0
1680.0	1500000.0	355.0	7300.0
W		FR	
EXPECTED VALUE (LBS)	VARIANCE (LBS2)	EXPECTED VALUE (FAILURES/MONTH)	VARIANCE (FAIL2/MN2)
215.0	2000.0	750.0	6000.0
225.0	2500.0	800.0	8000.0
250.0	3000.0	950.0	12000.0
360.0	3700.0	500.0	8000.0
375.0	4800.0	550.0	10000.0
425.0	5300.0	650.0	14000.0
450.0	7600.0	400.0	12000.0
500.0	9500.0	450.0	14000.0
600.0	13000.0	550.0	18000.0
MTIR		VARIANCE	
EXPECTED VALUE (MINUTES)			
120.0		100.0	
90.0		200.0	
50.0		300.0	
210.0		200.0	
150.0		350.0	
120.0		600.0	
300.0		800.0	
240.0		1500.0	
160.0		2500.0	

SUBSYSTEM 13

LOC	EXPECTED VALUE		VARIANCE		N	AND D	EXPECTED VALUE		VARIANCE	
	(\$X10-6)		(\$2X10-12)				(\$X10-6)		(\$2X10-12)	
	1400.0		40000.0				25.0		100.0	
	1305.0		60000.0				35.0		125.0	
	1330.0		80000.0				50.0		200.0	
	1340.0		200000.0				60.0		400.0	
	1305.0		230000.0				75.0		600.0	
	1300.0		435000.0				110.0		900.0	
	1320.0		60000.0				95.0		980.0	
	1308.0		1100000.0				100.0		950.0	
	1200.0		1310000.0				110.0		1000.0	
N	EXPECTED VALUE		VARIANCE		FR	EXPECTED VALUE	VARIANCE			
	(LMS)		(LMS2)			(FAILURES/MONTH)	(FAIL2/MN2)			
	70.0		200.0			400.0	500.0			
	80.0		400.0			420.0	700.0			
	90.0		900.0			450.0	1000.0			
	110.0		1000.0			200.0	700.0			
	130.0		1100.0			220.0	1000.0			
	170.0		1200.0			250.0	1600.0			
	200.0		2700.0			180.0	1000.0			
	250.0		4100.0			120.0	1300.0			
	330.0		6000.0			150.0	2000.0			
MTIR	EXPECTED VALUE		VARIANCE							
	(MINUTES)		(MIN2)							
	90.0		100.0							
	70.0		200.0							
	60.0		400.0							
	180.0		300.0							
	140.0		500.0							
	120.0		1200.0							
	240.0		600.0							
	200.0		1000.0							
	180.0		2500.0							

SUBSYSTEM 14

LCC	EXPECTED VALUE (X10-6)	VARIANCE (X10-12)	R AND D	EXPECTED VALUE (X10-6)	VARIANCE (X10-12)
	1500.0	40000.0		50.0	500.0
	1420.0	100000.0		125.0	1500.0
	1420.0	100000.0		125.0	1500.0
	1490.0	270000.0		150.0	2000.0
	1300.0	400000.0		250.0	5000.0
	1300.0	400000.0		250.0	5000.0
	1240.0	380000.0		255.0	4000.0
	1200.0	1510000.0		280.0	6000.0
	1200.0	1510000.0		280.0	6000.0
N	EXPECTED VALUE (LHS)	VARIANCE (LHS2)	FR	EXPECTED VALUE (FAILURES/MONTH)	VARIANCE (FAIL2/MN2)
	150.0	1500.0		300.0	250.0
	200.0	2010.0		350.0	500.0
	200.0	2000.0		350.0	500.0
	250.0	2500.0		200.0	500.0
	350.0	5050.0		250.0	1000.0
	350.0	5050.0		250.0	1000.0
	375.0	6100.0		100.0	1000.0
	450.0	10000.0		150.0	2000.0
	450.0	10000.0		150.0	2000.0
	EXPECTED VALUE (MINUTES)	VARIANCE (MIN2)		EXPECTED VALUE (MINUTES)	VARIANCE (MIN2)
	300.0	1000.0		1000.0	500.0
	180.0	3600.0		1000.0	500.0
	180.0	3600.0		1000.0	500.0
	360.0	1700.0		1000.0	500.0
	240.0	5100.0		1000.0	500.0
	240.0	5100.0		1000.0	500.0
	340.0	7200.0		1000.0	500.0
	420.0	7200.0		1000.0	500.0
	420.0	7200.0		1000.0	500.0

SUBSYSTEM 15

LCC EXPECTED VALUE		K AND D		EXPECTED VALUE		VARIANCE	
(S2X10-6)		(S2X10-6)		(S2X10-6)		(S2X10-12)	
1720.0	50000.0	50.0		50.0		1160.0	
1695.0	60000.0	80.0		80.0		1250.0	
1660.0	80000.0	140.0		140.0		1400.0	
1640.0	200000.0	150.0		150.0		1600.0	
1620.0	250000.0	175.0		175.0		1900.0	
1615.0	300000.0	200.0		200.0		2900.0	
1550.0	730000.0	290.0		290.0		3000.0	
1500.0	1400000.0	300.0		300.0		4000.0	
1450.0	1600000.0	330.0		330.0		6100.0	
W EXPECTED VALUE		FR		EXPECTED VALUE		VARIANCE	
(LHS)		(FAILURES/MONTH)		(FAIL2/MN2)		(FAIL2/MN2)	
125.0	1600.0	800.0		800.0		1000.0	
140.0	2500.0	900.0		900.0		2000.0	
170.0	3250.0	1050.0		1050.0		5000.0	
200.0	4490.0	600.0		600.0		3000.0	
230.0	5930.0	700.0		700.0		4000.0	
270.0	7500.0	900.0		900.0		7000.0	
300.0	7500.0	250.0		250.0		5000.0	
350.0	8500.0	300.0		300.0		8500.0	
400.0	10000.0	450.0		450.0		15000.0	
MTIR		EXPECTED VALUE		VARIANCE			
		(MINUTES)		(MIN2)			
		240.0		600.0			
		180.0		600.0			
		140.0		1200.0			
		260.0		750.0			
		210.0		950.0			
		170.0		1400.0			
		280.0		900.0			
		220.0		1100.0			
		160.0		1600.0			

SUBSYSTEM 16

LCC	EXPECTED VALUE (SXI0-6)	VARIANCE (SXI0-12)	K AND D	EXPECTED VALUE (SXI0-6)	VARIANCE (SXI0-12)
	1500.0	20000.0		60.0	1250.0
	1400.0	40000.0		75.0	1400.0
	1400.0	60000.0		125.0	1600.0
	1400.0	130000.0		150.0	1800.0
	1400.0	200000.0		200.0	2400.0
	1400.0	355000.0		250.0	3200.0
	1350.0	780000.0		215.0	3000.0
	1305.0	1200000.0		230.5	3250.0
	1200.0	1500000.0		260.0	4000.0
K	EXPECTED VALUE (LBS)	VARIANCE (LHS2)	FR	EXPECTED VALUE (FAILURES/MIN)	VARIANCE (FAIL2/HN2)
	60.0	400.0		560.0	1000.0
	70.0	450.0		600.0	2500.0
	80.0	530.0		700.0	5000.0
	90.0	600.0		350.0	2500.0
	100.0	650.0		400.0	4000.0
	115.0	700.0		500.0	7000.0
	150.0	1000.0		175.0	4000.0
	165.0	1300.0		230.0	6000.0
	190.0	2000.0		330.0	10000.0
LX	EXPECTED VALUE (MINUTES)	VARIANCE (MIN2)	MTIR	EXPECTED VALUE (MINUTES)	VARIANCE (MIN2)
	480.0	2000.0		2000.0	2000.0
	560.0	4000.0		4000.0	4000.0
	500.0	6000.0		6000.0	6000.0
	540.0	2400.0		2400.0	2400.0
	420.0	4500.0		4500.0	4500.0
	560.0	7200.0		7200.0	7200.0
	600.0	3000.0		3000.0	3000.0
	480.0	6000.0		6000.0	6000.0
	420.0	10000.0		10000.0	10000.0

SUBSYSTEM 17

LCC	EXPECTED VALUE		VARIANCE		R AND D		EXPECTED VALUE		VARIANCE	
	(SXI0-6)	(S2X10-12)	(SXI0-6)	(S2X10-12)	(SXI0-6)	(S2X10-12)	(SXI0-6)	(S2X10-12)	(SXI0-6)	(S2X10-12)
	2500.0	30000.0			70.0		70.0		848.0	
	2500.0	30000.0			70.0		70.0		848.0	
	2500.0	30000.0			70.0		70.0		848.0	
	2100.0	26000.0			175.0		175.0		2000.0	
	2100.0	26000.0			175.8		175.8		2000.0	
	2100.0	26000.0			175.8		175.8		2000.0	
	1800.0	125000.0			350.0		350.0		7600.0	
	1800.0	125000.0			350.0		350.0		7600.0	
	1800.0	125000.0			350.0		350.0		7600.0	
W	EXPECTED VALUE		VARIANCE		FR		EXPECTED VALUE		VARIANCE	
	(LHS)	(LBS2)	(LHS)	(LBS2)	(FAILURES/MONTH)	(FAIL2/MN2)	(FAILURES/MONTH)	(FAIL2/MN2)	(FAILURES/MONTH)	(FAIL2/MN2)
	150.0	2500.0			600.0		600.0		500.0	
	150.0	2500.0			600.0		600.0		500.0	
	150.0	2500.0			600.0		600.0		500.0	
	260.0	4750.0			300.0		300.0		1000.0	
	260.0	4750.0			300.0		300.0		1000.0	
	260.0	4750.0			300.0		300.0		1000.0	
	260.0	4750.0			150.0		150.0		2000.0	
	260.0	4750.0			150.0		150.0		2000.0	
	260.0	4750.0			150.0		150.0		2000.0	
MTIR	EXPECTED VALUE		VARIANCE		MTIR		EXPECTED VALUE		VARIANCE	
	(MINUTES)	(MIN2)	(MINUTES)	(MIN2)	(MINUTES)	(MIN2)	(MINUTES)	(MIN2)	(MINUTES)	(MIN2)
	180.0	2500.0			180.0		180.0		2500.0	
	180.0	2500.0			180.0		180.0		2500.0	
	180.0	2500.0			180.0		180.0		2500.0	
	210.0	3600.0			210.0		210.0		3600.0	
	210.0	3600.0			210.0		210.0		3600.0	
	210.0	3600.0			210.0		210.0		3600.0	
	240.0	4900.0			240.0		240.0		4900.0	
	240.0	4900.0			240.0		240.0		4900.0	
	240.0	4900.0			240.0		240.0		4900.0	

SUBSYSTEM 18

LCC		K AND D		VARIANCE	
EXPECTED VALUE (1X10-6)	(12X10-12)	EXPECTED VALUE (5X10-6)	(52X10-12)	VARIANCE	
1300.0	20000.0	37.5	170.0		
1285.0	40000.0	50.0	300.0		
1260.0	60000.0	75.0	600.0		
1270.0	130000.0	75.0	1000.0		
1250.0	160000.0	100.0	1100.0		
1245.0	265000.0	150.0	1800.0		
1150.0	600000.0	165.0	1500.0		
1150.0	900000.0	175.0	1700.0		
1140.0	1200000.0	200.0	2100.0		
W		FK		VARIANCE	
EXPECTED VALUE (LHS)	(LHS2)	EXPECTED VALUE (FAILURES/MONTH)	(FAIL2/MN2)	VARIANCE	
50.0	300.0	750.0	1000.0		
60.0	400.0	800.0	2000.0		
75.0	650.0	900.0	4000.0		
90.0	910.0	500.0	4000.0		
120.0	1500.0	550.0	5000.0		
155.0	2704.0	650.0	6000.0		
180.0	3605.0	250.0	6000.0		
195.0	4225.0	300.0	7000.0		
250.0	5111.0	400.0	11000.0		
MTTR		VARIANCE			
EXPECTED VALUE (MINUTES)	(MIN2)	EXPECTED VALUE (MIN2)	VARIANCE		
100.0	75.0				
70.0	100.0				
54.0	200.0				
160.0	90.0				
115.0	175.0				
90.0	350.0				
210.0	120.0				
150.0	200.0				
105.0	600.0				

LOG	EXPECTED VALUE (L1K10-6)	VARIANCE (L2K10-12)	N AND U	EXPECTED VALUE (L3K10-6)	VARIANCE (L2K10-12)
	2300.0	60000.0		25.0	160.0
	2315.0	150000.0		50.0	400.0
	2315.0	150000.0		50.0	400.0
	1960.0	500000.0		125.0	1600.0
	1930.0	1840000.0		210.0	3000.0
	1930.0	1840000.0		210.0	3000.0
	1980.0	500000.0		125.0	1600.0
	1930.0	1840000.0		210.0	3000.0
	1930.0	1840000.0		210.0	3000.0
M	EXPECTED VALUE (L8S)	VARIANCE (L8S2)	FK	EXPECTED VALUE (FAILURES/MONTH)	VARIANCE (FAIL2/MN2)
	75.0	800.0		800.0	5000.0
	95.0	1025.0		1000.0	10000.0
	95.0	1025.0		1000.0	10000.0
	100.0	2610.0		440.0	10000.0
	240.0	5000.0		560.0	20000.0
	240.0	5000.0		560.0	20000.0
	100.0	2610.0		440.0	10000.0
	240.0	5000.0		560.0	20000.0
	240.0	5000.0		560.0	20000.0
	MYR	EXPECTED VALUE (MINUTES)	VARIANCE (MIN2)		
		270.0	1000.0		
		210.0	4900.0		
		210.0	4900.0		
		350.0	3000.0		
		290.0	6500.0		
		290.0	6500.0		
		350.0	3000.0		
		290.0	6500.0		
		290.0	6500.0		

SUBSYSTEM 20

LOC	EXPECTED VALUE		VARIANCE		K	AND	D	EXPECTED VALUE		VARIANCE	
	(S10-6)	(S210-12)	(S10-6)	(S210-12)				(S10-6)	(S210-12)	(S10-6)	(S210-12)
	1200.0	5000.0	5000.0	500.0				50.0	500.0		
	1250.0	6000.0	6000.0	75.0				75.0	700.0		
	1200.0	10000.0	10000.0	125.0				125.0	800.0		
	1220.0	23000.0	23000.0	100.0				100.0	1400.0		
	1200.0	34000.0	34000.0	150.0				150.0	1700.0		
	1100.0	50500.0	50500.0	180.0				180.0	2000.0		
	1175.0	58000.0	58000.0	175.0				175.0	3000.0		
	1125.0	70000.0	70000.0	200.0				200.0	3250.0		
	1095.0	91000.0	91000.0	250.0				250.0	4100.0		
K	EXPECTED VALUE		VARIANCE		FK	EXPECTED VALUE	VARIANCE				
	(LHS)	(LHS2)	(LHS2)	(LHS2)				(FAILURES/MONTH)	(FAIL2/MN2)	(FAIL2/MN2)	(FAIL2/MN2)
	85.0	900.0	900.0	600.0				600.0	500.0		
	100.0	1000.0	1000.0	600.0				600.0	1000.0		
	125.0	1764.0	1764.0	750.0				750.0	2000.0		
	147.0	2401.0	2401.0	400.0				400.0	2000.0		
	170.0	3000.0	3000.0	450.0				450.0	3000.0		
	200.0	3300.0	3300.0	525.0				525.0	6000.0		
	230.0	3500.0	3500.0	200.0				200.0	4000.0		
	240.0	3600.0	3600.0	250.0				250.0	6000.0		
	250.0	4000.0	4000.0	350.0				350.0	10000.0		
MTTR	EXPECTED VALUE		VARIANCE		MTTR	EXPECTED VALUE	VARIANCE				
	(MINUTES)	(MIN2)	(MIN2)	(MIN2)				(MINUTES)	(MIN2)	(MIN2)	(MIN2)
	60.0	25.0	25.0	25.0				25.0			
	45.0	50.0	50.0	50.0				50.0			
	30.0	100.0	100.0	100.0				100.0			
	90.0	75.0	75.0	75.0				75.0			
	75.0	150.0	150.0	150.0				150.0			
	60.0	300.0	300.0	300.0				300.0			
	150.0	200.0	200.0	200.0				200.0			
	135.0	400.0	400.0	400.0				400.0			
	120.0	900.0	900.0	900.0				900.0			

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